$\because V = E.l$ and $\frac{I}{A} = j$

by Eq (3.10) we get

$$E\,l = j\,\rho\,l$$

or, $\boxed{E = j\,\rho}$ (3.11)

The above relation for ***magnitudes E*** and $j$ can indeed be cast in a ***vector*** form. The current density, (which we have defined as the current through unit area ***normal*** to the current) is also directed along **E**, and is also a vector **j** ($\equiv j\,\mathbf{E}/E$). Thus, the last equation can be written as,

$$\mathbf{E} = \mathbf{j}\rho \qquad (3.12)$$

or, $\boxed{\mathbf{j} = \sigma\,\mathbf{E}}$ $\left(\because \sigma = \frac{1}{\rho}\right)$ (3.13)

where $\sigma \equiv 1/\rho$ is called the *conductivity*. Ohm's law is often stated in an equivalent form, Eq. (3.13) in addition to Eq.(3.3). In the next section, we will try to understand the origin of the Ohm's law as arising from the characteristics of the drift of electrons.

## 3.5 Drift of Electrons and the Origin of Resistivity

As remarked before, an electron will suffer collisions with the heavy fixed ions, but after collision, it will emerge with the same speed but in random directions. If we consider all the electrons, their average velocity will be zero since their directions are random. Thus, if there are $N$ electrons and the velocity of the $i^{th}$ electron ($i$ = 1, 2, 3, ... $N$) at a given time is $\mathbf{v}_i$, then

In absence of electric field inside the conductor

$$\frac{1}{N}\sum_{i=1}^{N}\mathbf{v}_i = 0 \qquad (3.14)$$

Consider now the situation when an electric field is present. Electrons will be accelerated due to this field by

$$\mathbf{a} = \frac{-e\,\mathbf{E}}{m} \qquad (3.15)$$

where $-e$ is the charge and $m$ is the mass of an electron. Consider again the $i^{th}$ electron at a given time $t$. This electron would have had its last collision some time before $t$, and let $t_i$ be the time elapsed after its last collision. If $\mathbf{v}_i$ was its velocity immediately after the last collision, then its velocity $\mathbf{V}_i$ at time $t$ is

$$\mathbf{V}_i = \mathbf{v}_i + \frac{-e\mathbf{E}}{m}\,t_i \qquad (3.16)$$

(formula $\vec{v} = \vec{u} + \vec{a}t$ is applied)

since starting with its last collision it was accelerated (Fig. 3.3) with an acceleration given by Eq. (3.15) for a time interval $t_i$. The average velocity of the electrons at time $t$ is the average of all the $\mathbf{V}_i$'s. The average of $\mathbf{v}_i$'s is zero [Eq. (3.14)] since immediately after any collision, the direction of the velocity of an electron is completely random. The collisions of the electrons do not occur at regular intervals but at random times. Let us denote by $\tau$, the average time between successive collisions. Then at a given time, some of the electrons would have spent

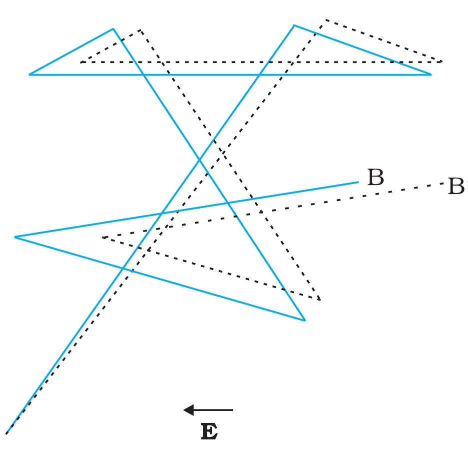


**FIGURE 3.3** A schematic picture of an electron moving from a point A to another point B through repeated collisions, and straight line travel between collisions (full lines). If an electric field is applied as shown, the electron ends up at point B′ (dotted lines). A slight drift in a direction opposite the electric field is visible.

$\tau$ = relaxation time