

Example 1.4 Coulomb's law for electrostatic force between two point charges and Newton's law for gravitational force between two stationary point masses, both have inverse-square dependence on the distance between the charges and masses respectively. (a) Compare the strength of these forces by determining the ratio of their magnitudes (i) for an electron and a proton and (ii) for two protons. (b) Estimate the accelerations of electron and proton due to the electrical force of their mutual attraction when they are $1 \text{ \AA} (= 10^{-10} \text{ m})$ apart? ($m_p = 1.67 \times 10^{-27} \text{ kg}$, $m_e = 9.11 \times 10^{-31} \text{ kg}$)

Solution

(a) (i) The electric force between an electron and a proton at a distance r apart is:

$$F_e = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

where the negative sign indicates that the force is attractive. The corresponding gravitational force (always attractive) is:

$$F_G = -G \frac{m_p m_e}{r^2}$$

where m_p and m_e are the masses of a proton and an electron respectively.

$$\left| \frac{F_e}{F_G} \right| = \frac{e^2}{4\pi\epsilon_0 G m_p m_e} = 2.4 \times 10^{39}$$

(ii) On similar lines, the ratio of the magnitudes of electric force to the gravitational force between two protons at a distance r apart is:

$$\left| \frac{F_e}{F_G} \right| = \frac{e^2}{4\pi\epsilon_0 G m_p m_p} = 1.3 \times 10^{36}$$

However, it may be mentioned here that the signs of the two forces are different. For two protons, the gravitational force is attractive in nature and the Coulomb force is repulsive. The actual values of these forces between two protons inside a nucleus (distance between two protons is $\sim 10^{-15} \text{ m}$ inside a nucleus) are $F_e \sim 230 \text{ N}$, whereas, $F_G \sim 1.9 \times 10^{-34} \text{ N}$.

The (dimensionless) ratio of the two forces shows that electrical forces are enormously stronger than the gravitational forces.

(b) The electric force \mathbf{F} exerted by a proton on an electron is same in magnitude to the force exerted by an electron on a proton; however, the masses of an electron and a proton are different. Thus, the magnitude of force is

$$\begin{aligned} |\mathbf{F}| &= \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = 8.987 \times 10^9 \text{ Nm}^2/\text{C}^2 \times (1.6 \times 10^{-19} \text{ C})^2 / (10^{-10} \text{ m})^2 \\ &= 2.3 \times 10^{-8} \text{ N} \end{aligned}$$

Using Newton's second law of motion, $F = ma$, the acceleration that an electron will undergo is

$$a = 2.3 \times 10^{-8} \text{ N} / 9.11 \times 10^{-31} \text{ kg} = 2.5 \times 10^{22} \text{ m/s}^2$$

Comparing this with the value of acceleration due to gravity, we can conclude that the effect of gravitational field is negligible on the motion of electron and it undergoes very large accelerations under the action of Coulomb force due to a proton.

The value for acceleration of the proton is

$$2.3 \times 10^{-8} \text{ N} / 1.67 \times 10^{-27} \text{ kg} = 1.4 \times 10^{19} \text{ m/s}^2$$

PHYSICS

Interactive animation on Coulomb's law:
http://webphysics.davidson.edu/physlet_resources/bu_semester2/menu_semester2.html

EXAMPLE 1.4