test of consistency of units, but has the advantage that we need not commit ourselves to a particular choice of units, and we need not worry about conversions among multiples and sub-multiples of the units. It may be borne in mind that **if an equation fails this consistency test, it is proved wrong, but if it passes, it is not proved right. Thus, a dimensionally correct equation need not be actually an exact (correct) equation, but a dimensionally wrong (incorrect) or inconsistent equation must be wrong**.

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Example 2.15 Let us consider an equation

$$\frac{1}{2} m\,v^2 = m\,g\,h$$

where m is the mass of the body, v its velocity, g is the acceleration due to gravity and h is the height. Check whether this equation is dimensionally correct.

Answer The dimensions of LHS are

$$[M]\ [L\,T^{-1}]^2 = [M]\,[L^2 T^{-2}] = [M\,L^2\,T^{-2}]$$

The dimensions of RHS are

$$[M][L\,T^{-2}]\ [L] = [M][L^2\,T^{-2}] = [M\,L^2\,T^{-2}]$$

The dimensions of LHS and RHS are the same and hence the equation is dimensionally correct. ◀

Example 2.16 The SI unit of energy is J = kg $m^2 s^{-2}$; that of speed v is m s^{-1} and of acceleration a is m s^{-2}. Which of the formulae for kinetic energy (K) given below can you rule out on the basis of dimensional arguments (m stands for the mass of the body) :

(a) $K = m^2 v^3$
(b) $K = (1/2)mv^2$
(c) $K = ma$
(d) $K = (3/16)mv^2$
(e) $K = (1/2)mv^2 + ma$

Answer Every correct formula or equation must have the same dimensions on both sides of the equation. Also, only quantities with the same physical dimensions can be added or subtracted. The dimensions of the quantity on the right side are $[M^2\,L^3\,T^{-3}]$ for (a); $[M\,L^2\,T^{-2}]$ for (b) and (d); $[M\,L\,T^{-2}]$ for (c). The quantity on the right side of (e) has no proper dimensions since two quantities of different dimensions have been added. Since the kinetic energy K has the dimensions of $[M\,L^2\,T^{-2}]$, formulas (a), (c) and (e) are ruled out. Note that dimensional arguments cannot tell which of the two, (b) or (d), is the correct formula. For this, one must turn to the actual definition of kinetic energy (see Chapter 6). The correct formula for kinetic energy is given by (b). ◀

2.10.2 Deducing Relation among the Physical Quantities

The method of dimensions can sometimes be used to deduce relation among the physical quantities. For this we should know the dependence of the physical quantity on other quantities (upto three physical quantities or linearly independent variables) and consider it as a product type of the dependence. Let us take an example.

Example 2.17 Consider a simple pendulum, having a bob attached to a string, that oscillates under the action of the force of gravity. Suppose that the period of oscillation of the simple pendulum depends on its length (l), mass of the bob (m) and acceleration due to gravity (g). Derive the expression for its time period using method of dimensions.

Answer The dependence of time period T on the quantities l, g and m as a product may be written as :

$$T = k\, l^x\, g^y\, m^z$$

where k is dimensionless constant and x, y and z are the exponents.

By considering dimensions on both sides, we have

$$[L^o M^o T^1] = [L^1]^x\,[L^1 T^{-2}]^y\,[M^1]^z$$

$$= L^{x+y}\,T^{-2y}\,M^z$$

On equating the dimensions on both sides, we have

$$x + y = 0;\ -2y = 1;\ \text{and}\ z = 0$$

So that $x = \frac{1}{2}, y = -\frac{1}{2}, z = 0$

Then, $T = k\, l^{½}\, g^{-½}$