quantity. Thus, the dimensional equations are the equations, which represent the dimensions of a physical quantity in terms of the base quantities. For example, the dimensional equations of volume $[V]$, speed $[\upsilon]$, force $[F]$ and mass density $[\rho]$ may be expressed as

$$[V] = [M^0\,L^3\,T^0]$$
$$[\upsilon] = [M^0\,L\,T^{-1}]$$
$$[F] = [M\,L\,T^{-2}]$$
$$[\rho] = [M\,L^{-3}\,T^0]$$

The dimensional equation can be obtained from the equation representing the relations between the physical quantities. The dimensional formulae of a large number and wide variety of physical quantities, derived from the equations representing the relationships among other physical quantities and expressed in terms of base quantities are given in Appendix 9 for your guidance and ready reference.

2.10 DIMENSIONAL ANALYSIS AND ITS APPLICATIONS

Note 1

The recognition of concepts of dimensions, which guide the description of physical behaviour is of basic importance as only those physical quantities can be added or subtracted which have the same dimensions. A thorough understanding of dimensional analysis helps us in deducing certain relations among different physical quantities and checking the derivation, accuracy and dimensional consistency or homogeneity of various mathematical expressions. When magnitudes of two or more physical quantities are multiplied, their units should be treated in the same manner as ordinary algebraic symbols. We can cancel identical units in the numerator and denominator. The same is true for dimensions of a physical quantity. Similarly, physical quantities represented by symbols on both sides of a mathematical equation must have the same dimensions.

Note 2

Note3

2.10.1 Checking the Dimensional Consistency of Equations

The magnitudes of physical quantities may be added together or subtracted from one another only if they have the same dimensions. In other words, we can add or subtract similar physical quantities. Thus, velocity cannot be added to force, or an electric current cannot be subtracted from the thermodynamic temperature. This simple principle called **the principle of homogeneity of dimensions** in an equation is extremely useful in checking the correctness of an equation. If the dimensions of all the terms are not same, the equation is wrong. Hence, if we derive an expression for the length (or distance) of an object, regardless of the symbols appearing in the original mathematical relation, when all the individual dimensions are simplified, the remaining dimension must be that of length. Similarly, if we derive an equation of speed, the dimensions on both the sides of equation, when simplified, must be of length/time, or $[L\,T^{-1}]$.

Dimensions are customarily used as a preliminary test of the consistency of an equation, when there is some doubt about the correctness of the equation. However, the dimensional consistency does not guarantee correct equations. It is uncertain to the extent of dimensionless quantities or functions. The arguments of special functions, such as the trigonometric, logarithmic and exponential functions must be dimensionless. A pure number, ratio of similar physical quantities, such as angle as the ratio (length/length), refractive index as the ratio (speed of light in vacuum/speed of light in medium) etc., has no dimensions.

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Now we can test the dimensional consistency or homogeneity of the equation

$$x = x_0 + \upsilon_0\,t + (1/2)\,a\,t^2$$

for the distance x travelled by a particle or body in time t which starts from the position x_0 with an initial velocity υ_0 at time $t = 0$ and has uniform acceleration a along the direction of motion.

The dimensions of each term may be written as

$$[x] = [L]$$
$$[x_0] = [L]$$
$$[\upsilon_0\,t] = [L\,T^{-1}]\ [T] = [L]$$
$$[(1/2)\,a\,t^2] = [L\,T^{-2}]\ [T^2] = [L]$$

As each term on the right hand side of this equation has the same dimension, namely that of length, which is same as the dimension of left hand side of the equation, hence this equation is a dimensionally correct equation.

It may be noted that a test of consistency of dimensions tells us no more and no less than a