Example 2.9 The resistance $R = V/I$ where $V = (100 \pm 5)$V and $I = (10 \pm 0.2)$A. Find the percentage error in R.

Answer The percentage error in V is 5% and in I it is 2%. The total error in R would therefore be 5% + 2% = 7%. ◀

Example 2.10 Two resistors of resistances $R_1 = 100 \pm 3$ ohm and $R_2 = 200 \pm 4$ ohm are connected (a) in series, (b) in parallel. Find the equivalent resistance of the (a) series combination, (b) parallel combination. Use for (a) the relation $R = R_1 + R_2$, and for (b) $\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$ and $\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$

Answer (a) The equivalent resistance of series combination

$R = R_1 + R_2 = (100 \pm 3)$ ohm + (200 ± 4) ohm

$= 300 \pm 7$ ohm.

(b) The equivalent resistance of parallel combination

$$R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{200}{3} = 66.7 \text{ ohm}$$

Then, from $\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$

we get,

$$\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\Delta R' = \left(R'^2\right)\frac{\Delta R_1}{R_1^2} + \left(R'^2\right)\frac{\Delta R_2}{R_2^2}$$

$$= \left(\frac{66.7}{100}\right)^2 3 + \left(\frac{66.7}{200}\right)^2 4$$

$= 1.8$

Then, $R' = 66.7 \pm 1.8$ ohm

(Here, ΔR is expresed as 1.8 instead of 2 to keep in confirmity with the rules of significant figures.) ◀

(c) Error in case of a measured quantity raised to a power

Suppose $Z = A^2$,

Then,

$\Delta Z/Z = (\Delta A/A) + (\Delta A/A) = 2\,(\Delta A/A)$.

Hence, the relative error in A^2 is two times the error in A.

In general, if $Z = A^p B^q / C^r$

Then,

$\Delta Z/Z = p\,(\Delta A/A) + q\,(\Delta B/B) + r\,(\Delta C/C)$.

Hence the rule : The relative error in a physical quantity raised to the power k is the k times the relative error in the individual quantity.

Example 2.11 Find the relative error in Z, if $Z = A^4 B^{1/3}/CD^{3/2}$.

Answer The relative error in Z is $\Delta Z/Z = 4(\Delta A/A) + (1/3)\,(\Delta B/B) + (\Delta C/C) + (3/2)\,(\Delta D/D)$. ◀

Example 2.12 The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{L/g}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. What is the accuracy in the determination of g ?

Answer $g = 4\pi^2 L/T^2$

Here, $T = \frac{t}{n}$ and $\Delta T = \frac{\Delta t}{n}$. Therefore, $\frac{\Delta T}{T} = \frac{\Delta t}{t}$.

The errors in both L and t are the least count errors. Therefore,

$(\Delta g/g) = (\Delta L/L) + 2(\Delta T/T)$

$$= \frac{0.1}{20.0} + 2\left(\frac{1}{90}\right) = 0.027$$

Thus, the percentage error in g is

$100\,(\Delta g/g) = 100(\Delta L/L) + 2 \times 100\,(\Delta T/T)$

$= 3\%$ ◀

2.7 SIGNIFICANT FIGURES

As discussed above, every measurement involves errors. Thus, the result of measurement should be reported in a way that indicates the precision of measurement. Normally, the reported result of measurement is a number that includes all digits in the number that are known reliably plus the first digit that is uncertain. The reliable digits plus

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