How will you measure the length of a line?

What a naïve question, at this stage, you might say! But what if it is not a straight line? Draw a zigzag line in your copy, or on the blackboard. Well, not too difficult again. You might take a thread, place it along the line, open up the thread, and measure its length.

Now imagine that you want to measure the length of a national highway, a river, the railway track between two stations, or the boundary between two states or two nations. If you take a string of length 1 metre or 100 metre, keep it along the line, shift its position every time, the arithmetic of man-hours of labour and expenses on the project is not commensurate with the outcome. Moreover, errors are bound to occur in this enormous task. There is an interesting fact about this. France and Belgium share a common international boundary, whose length mentioned in the official documents of the two countries differs substantially!

Go one step beyond and imagine the coastline where land meets sea. Roads and rivers have fairly mild bends as compared to a coastline. Even so, all documents, including our school books, contain information on the length of the coastline of Gujarat or Andhra Pradesh, or the common boundary between two states, etc. Railway tickets come with the distance between stations printed on them. We have 'milestones' all along the roads indicating the distances to various towns. So, how is it done?

One has to decide how much error one can tolerate and optimise cost-effectiveness. If you want smaller errors, it will involve high technology and high costs. Suffice it to say that it requires fairly advanced level of physics, mathematics, engineering and technology. It belongs to the areas of fractals, which has lately become popular in theoretical physics. Even then one doesn't know how much to rely on the figure that props up, as is clear from the story of France and Belgium. Incidentally, this story of the France-Belgium discrepancy appears on the first page of an advanced Physics book on the subject of fractals and chaos!

mass density is obtained by deviding mass by the volume of the substance. If we have errors in the measurement of mass and of the sizes or dimensions, we must know what the error will be in the density of the substance. To make such estimates, we should learn how errors combine in various mathematical operations. For this, we use the following procedure.

(a) *Error of a sum or a difference*

Suppose two physical quantities A and B have measured values $A \pm \Delta A$, $B \pm \Delta B$ respectively where ΔA and ΔB are their absolute errors. We wish to find the error ΔZ in the sum

$$Z = A + B.$$

We have by addition, $Z \pm \Delta Z$

$$= (A \pm \Delta A) + (B \pm \Delta B).$$

The maximum possible error in Z

$$\Delta Z = \Delta A + \Delta B$$

For the difference $Z = A - B$, we have

$$Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$$

$$= (A - B) \pm \Delta A \pm \Delta B$$

or, $\pm \Delta Z = \pm \Delta A \pm \Delta B$

The maximum value of the error ΔZ is again $\Delta A + \Delta B$.

Hence the rule : When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.

Example 2.8 The temperatures of two bodies measured by a thermometer are t_1 = 20 °C ± 0.5 °C and t_2 = 50 °C ± 0.5 °C. Calculate the temperature difference and the error theirin.

Answer $t' = t_2 - t_1$ = (50 °C±0.5 °C)– (20°C±0.5 °C)

t' = 30 °C ± 1 °C

(b) Error of a product or a quotient

Suppose $Z = AB$ and the measured values of A and B are $A \pm \Delta A$ and $B \pm \Delta B$. Then

$$Z \pm \Delta Z = (A \pm \Delta A)\ (B \pm \Delta B)$$

$$= AB \pm B\,\Delta A \pm A\,\Delta B \pm \Delta A\,\Delta B.$$

Dividing LHS by Z and RHS by AB we have,

$$1 \pm (\Delta Z/Z) = 1 \pm (\Delta A/A) \pm (\Delta B/B) \pm (\Delta A/A)(\Delta B/B).$$

Since ΔA and ΔB are small, we shall ignore their product.

Hence the maximum relative error

$\Delta Z/\ Z = (\Delta A/A) + (\Delta B/B)$.

You can easily verify that this is true for division also.

Hence the rule : When two quantities are multiplied or divided, the relative error in the result is the sum of the relative errors in the multipliers.