Relative error = $\Delta a_{mean}/a_{mean}$ (2.9)

Def. When the relative error is expressed in per cent, it is called the **percentage error** (δa).

Thus, Percentage error

$$\delta a = (\Delta a_{mean}/a_{mean}) \times 100\% \quad (2.10)$$

Let us now consider an example.

Example 2.6 Two clocks are being tested against a standard clock located in a national laboratory. At 12:00:00 noon by the standard clock, the readings of the two clocks are :

	Clock 1	Clock 2
Monday	12:00:05	10:15:06
Tuesday	12:01:15	10:14:59
Wednesday	11:59:08	10:15:18
Thursday	12:01:50	10:15:07
Friday	11:59:15	10:14:53
Saturday	12:01:30	10:15:24
Sunday	12:01:19	10:15:11

If you are doing an experiment that requires precision time interval measurements, which of the two clocks will you prefer ?

Answer The range of variation over the seven days of observations is 162 s for clock 1, and 31 s for clock 2. The average reading of clock 1 is much closer to the standard time than the average reading of clock 2. The important point is that a clock's zero error is not as significant for precision work as its variation, because a 'zero-error' can always be easily corrected. Hence clock 2 is to be preferred to clock 1.

Example 2.7 We measure the period of oscillation of a simple pendulum. In successive measurements, the readings turn out to be 2.63 s, 2.56 s, 2.42 s, 2.71s and 2.80 s. Calculate the absolute errors, relative error or percentage error.

Answer The mean period of oscillation of the pendulum

$$T = \frac{(2.63 + 2.56 + 2.42 + 2.71 + 2.80)\text{s}}{5}$$

$$= \frac{13.12}{5}\text{ s}$$

$$= 2.624 \text{ s}$$
$$= 2.62 \text{ s}$$

As the periods are measured to a resolution of 0.01 s, all times are to the second decimal; it is proper to put this mean period also to the second decimal.

The errors in the measurements are

2.63 s – 2.62 s = 0.01 s
2.56 s – 2.62 s = – 0.06 s
2.42 s – 2.62 s = – 0.20 s
2.71 s – 2.62 s = 0.09 s
2.80 s – 2.62 s = 0.18 s

Note that the errors have the same units as the quantity to be measured.

The arithmetic mean of all the absolute errors (for arithmetic mean, we take only the magnitudes) is

$$\Delta T_{mean} = [(0.01 + 0.06 + 0.20 + 0.09 + 0.18)\text{s}]/5$$
$$= 0.54 \text{ s}/5$$
$$= 0.11 \text{ s}$$

That means, the period of oscillation of the simple pendulum is (2.62 ± 0.11) s i.e. it lies between (2.62 + 0.11) s and (2.62 – 0.11) s or between 2.73 s and 2.51 s. As the arithmetic mean of all the absolute errors is 0.11 s, there is already an error in the tenth of a second. Hence there is no point in giving the period to a hundredth. A more correct way will be to write

$$T = 2.6 \pm 0.1 \text{ s}$$

Note that the last numeral 6 is unreliable, since it may be anything between 5 and 7. We indicate this by saying that the measurement has two significant figures. In this case, the two significant figures are 2, which is reliable and 6, which has an error associated with it. You will learn more about the significant figures in section 2.7.

Important concept ⇒ There should be only the 1st digit in the value of error

So, concept of significant figures has to be included in the result.

For this example, the relative error or the percentage error is

$$\delta a = \frac{0.1}{2.6} \times 100 = 4\%$$

2.6.2 Combination of Errors

If we do an experiment involving several measurements, we must know how the errors in all the measurements combine. For example,