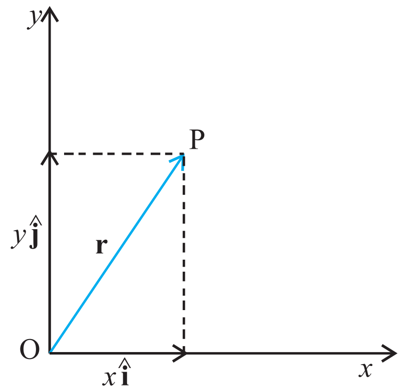
4.7.1 Position Vector and Displacement

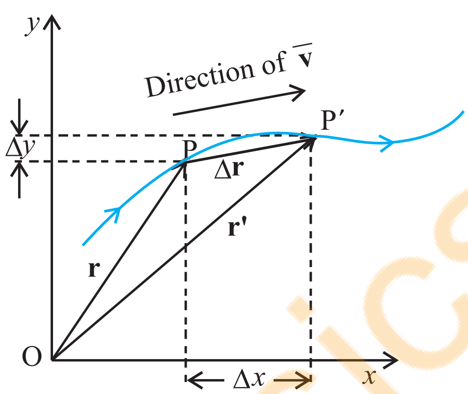
The position vector $\mathbf{r}$ of a particle P located in a plane with reference to the origin of an x-y reference frame (Fig. 4.12) is given by

$$\mathbf{r} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}$$

where x and y are components of $\mathbf{r}$ along x-, and y- axes or simply they are the coordinates of the object.



(a)



(b)

Fig. 4.12 (a) *Position vector* $\mathbf{r}$. (b) *Displacement* $\Delta\mathbf{r}$ *and average velocity* $\overline{\mathbf{v}}$ *of a particle.*

Suppose a particle moves along the curve shown by the thick line and is at P at time t and P′ at time t' [Fig. 4.12(b)]. Then, the displacement is :

$$\Delta\mathbf{r} = \mathbf{r}' - \mathbf{r} \qquad (4.25)$$

and is directed from P to P′ .

We can write Eq. (4.25) in a component form:

$$\Delta\mathbf{r} = \left(x'\,\hat{\mathbf{i}} + y'\,\hat{\mathbf{j}}\right) - \left(x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}\right)$$

$$= \hat{\mathbf{i}}\Delta x + \hat{\mathbf{j}}\Delta y$$

where $\quad \Delta x = x' - x,\ \Delta y = y' - y \qquad (4.26)$

Velocity

The average velocity $(\overline{\mathbf{v}})$ of an object is the ratio of the displacement and the corresponding time interval :

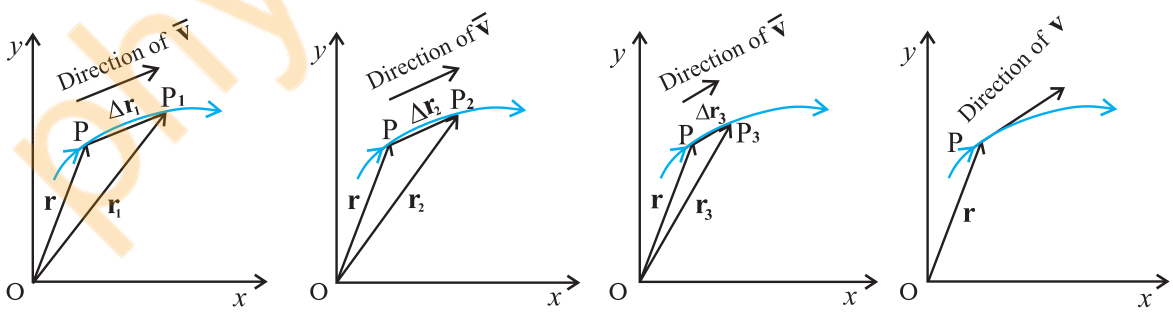
$$\overline{\mathbf{v}} = \frac{\Delta\mathbf{r}}{\Delta t} = \frac{\Delta x\,\hat{\mathbf{i}} + \Delta y\,\hat{\mathbf{j}}}{\Delta t} = \hat{\mathbf{i}}\frac{\Delta x}{\Delta t} + \hat{\mathbf{j}}\frac{\Delta y}{\Delta t} \qquad (4.27)$$

Or, $\quad \overline{\mathbf{v}} = \overline{v}_x\,\hat{\mathbf{i}} + \overline{v}_y\,\hat{\mathbf{j}}$

Since $\overline{\mathbf{v}} = \dfrac{\Delta\mathbf{r}}{\Delta t}$, the direction of the average velocity is the same as that of $\Delta\mathbf{r}$ (Fig. 4.12). The **velocity** (instantaneous velocity) is given by the limiting value of the average velocity as the time interval approaches zero :

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt} \qquad (4.28)$$

The meaning of the limiting process can be easily understood with the help of Fig 4.13(a) to (d). In these figures, the thick line represents the path of an object, which is at P at time t. P_1, P_2 and P_3 represent the positions of the object after times $\Delta t_1, \Delta t_2$, and Δt_3. $\Delta\mathbf{r}_1$, $\Delta\mathbf{r}_2$, and $\Delta\mathbf{r}_3$ are the displacements of the object in times Δt_1, Δt_2, and



(a) (b) (c) (d)

Fig. 4.13 *As the time interval* Δt *approaches zero, the average velocity approaches the velocity* $\mathbf{v}$. *The direction of* $\overline{\mathbf{v}}$ *is parallel to the line tangent to the path.*