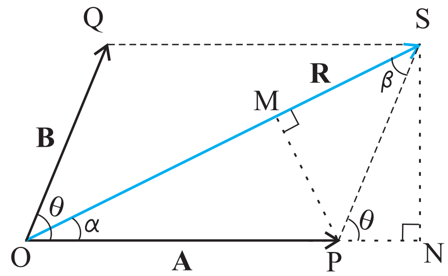

Fig. 4.10

Answer Let **OP** and **OQ** represent the two vectors **A** and **B** making an angle θ (Fig. 4.10). Then, using the parallelogram method of vector addition, **OS** represents the resultant vector **R** :

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

SN is normal to *OP* and *PM* is normal to *OS*.

From the geometry of the figure,

$$OS^2 = ON^2 + SN^2$$

but $ON = OP + PN = A + B\cos\theta$

$SN = B\sin\theta$

$$OS^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$

or, $R^2 = A^2 + B^2 + 2AB\cos\theta$

Law of cosines

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} \qquad (4.24a)$$

In Δ OSN, $SN = OS\sin\alpha = R\sin\alpha$, and

in Δ PSN, $SN = PS\sin\theta = B\sin\theta$

Therefore, $R\sin\alpha = B\sin\theta$

$$\text{or,} \quad \frac{R}{\sin\theta} = \frac{B}{\sin\alpha} \qquad (4.24b)$$

Similarly,

$$PM = A\sin\alpha = B\sin\beta$$

$$\text{or,} \quad \frac{A}{\sin\beta} = \frac{B}{\sin\alpha} \qquad (4.24c)$$

Combining Eqs. (4.24b) and (4.24c), we get

Law of sines

$$\boxed{\frac{R}{\sin\theta} = \frac{A}{\sin\beta} = \frac{B}{\sin\alpha}} \qquad (4.24d)$$

Using Eq. (4.24d), we get:

$$\sin\alpha = \frac{B}{R}\sin\theta \qquad (4.24e)$$

where *R* is given by Eq. (4.24a).

$$\text{or,} \quad \tan\alpha = \frac{SN}{OP + PN} = \frac{B\sin\theta}{A + B\cos\theta} \qquad (4.24f)$$

Equation (4.24a) gives the magnitude of the resultant and Eqs. (4.24e) and (4.24f) its direction. Equation (4.24a) is known as the **Law of cosines** and Eq. (4.24d) as the **Law of sines**.

Imp

Example 4.3 A motorboat is racing towards north at 25 km/h and the water current in that region is 10 km/h in the direction of 60° east of south. Find the resultant velocity of the boat.

Answer The vector $\mathbf{v}_b$ representing the velocity of the motorboat and the vector $\mathbf{v}_c$ representing the water current are shown in Fig. 4.11 in directions specified by the problem. Using the parallelogram method of addition, the resultant **R** is obtained in the direction shown in the figure.

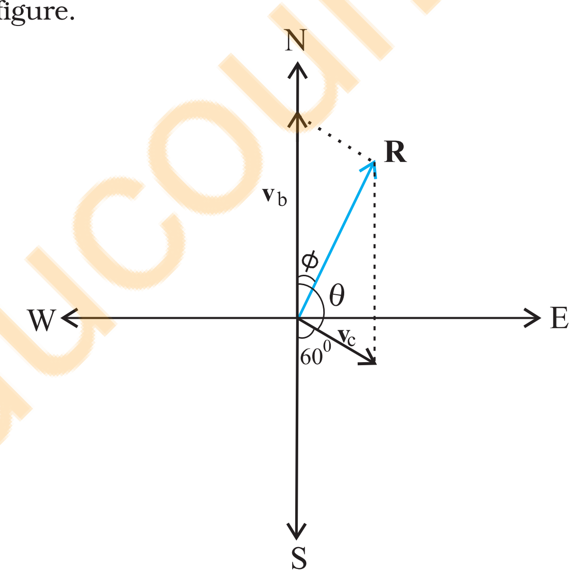

Fig. 4.11

We can obtain the magnitude of **R** using the Law of cosine :

$$R = \sqrt{v_b^2 + v_c^2 + 2v_b v_c \cos 120°}$$

$$= \sqrt{25^2 + 10^2 + 2\times25\times10(-1/2)} \cong 22 \text{ km/h}$$

To obtain the direction, we apply the Law of sines

$$\frac{R}{\sin\theta} = \frac{v_c}{\sin\phi} \quad \text{or,} \quad \sin\phi = \frac{v_c}{R}\sin\theta$$

$$= \frac{10\times\sin120^\circ}{21.8} = \frac{10\sqrt{3}}{2\times21.8} \cong 0.397$$

$$\phi \cong 23.4^\circ$$

4.7 MOTION IN A PLANE

In this section we shall see how to describe motion in two dimensions using vectors.