a given vector into two component vectors along a set of two vectors – all the three lie in the same plane. It is convenient to resolve a general vector along the axes of a rectangular coordinate system using vectors of unit magnitude. These are called unit vectors that we discuss now. A unit vector is a vector of unit magnitude and points in a particular direction. It has no dimension and unit. It is used to specify a direction only. Unit vectors along the x-, y- and z-axes of a rectangular coordinate system are denoted by $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$, respectively, as shown in Fig. 4.9(a).

Since these are unit vectors, we have

$$|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1 \qquad (4.9)$$

These unit vectors are perpendicular to each other. In this text, they are printed in bold face with a cap (^) to distinguish them from other vectors. Since we are dealing with motion in two dimensions in this chapter, we require use of only two unit vectors. If we multiply a unit vector, say $\hat{\mathbf{n}}$ by a scalar, the result is a vector $\boldsymbol{\lambda} = \lambda\hat{\mathbf{n}}$. In general, a vector **A** can be written as

$$\mathbf{A} = |\mathbf{A}|\,\hat{\mathbf{n}} \qquad (4.10)$$

where $\hat{\mathbf{n}}$ is a unit vector along **A**.

We can now resolve a vector **A** in terms of component vectors that lie along unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$. Consider a vector **A** that lies in x-y plane as shown in Fig. 4.9(b). We draw lines from the head of **A** perpendicular to the coordinate axes as in Fig. 4.9(b), and get vectors $\mathbf{A_1}$ and $\mathbf{A_2}$ such that $\mathbf{A_1} + \mathbf{A_2} = \mathbf{A}$. Since $\mathbf{A_1}$ is parallel to $\hat{\mathbf{i}}$ and $\mathbf{A_2}$ is parallel to $\hat{\mathbf{j}}$, we have :

$$\mathbf{A_1} = A_x\,\hat{\mathbf{i}}, \quad \mathbf{A_2} = A_y\,\hat{\mathbf{j}} \qquad (4.11)$$

where A_x and A_y are real numbers.

Thus, $$\mathbf{A} = A_x\,\hat{\mathbf{i}} + A_y\,\hat{\mathbf{j}} \qquad (4.12)$$

This is represented in Fig. 4.9(c). The quantities A_x and A_y are called x-, and y- components of the vector **A**. Note that A_x is itself not a vector, but $A_x\,\hat{\mathbf{i}}$ is a vector, and so is $A_y\,\hat{\mathbf{j}}$. Using simple trigonometry, we can express A_x and A_y in terms of the magnitude of **A** and the angle θ it makes with the x-axis :

$$A_x = A\cos\theta$$
$$A_y = A\sin\theta \qquad (4.13)$$

As is clear from Eq. (4.13), a component of a vector can be positive, negative or zero depending on the value of θ.

Now, we have two ways to specify a vector **A** in a plane. It can be specified by :

(i) its magnitude A and the direction θ it makes with the x-axis; or

(ii) its components A_x and A_y

If A and θ are given, A_x and A_y can be obtained using Eq. (4.13). If A_x and A_y are given, A and θ can be obtained as follows :

$$A_x^2 + A_y^2 = A^2\cos^2\theta + A^2\sin^2\theta$$
$$= A^2$$

Or, $$A = \sqrt{A_x^2 + A_y^2} \qquad (4.14)$$

And $$\tan\theta = \frac{A_y}{A_x}, \quad \theta = \tan^{-1}\frac{A_y}{A_x} \qquad (4.15)$$

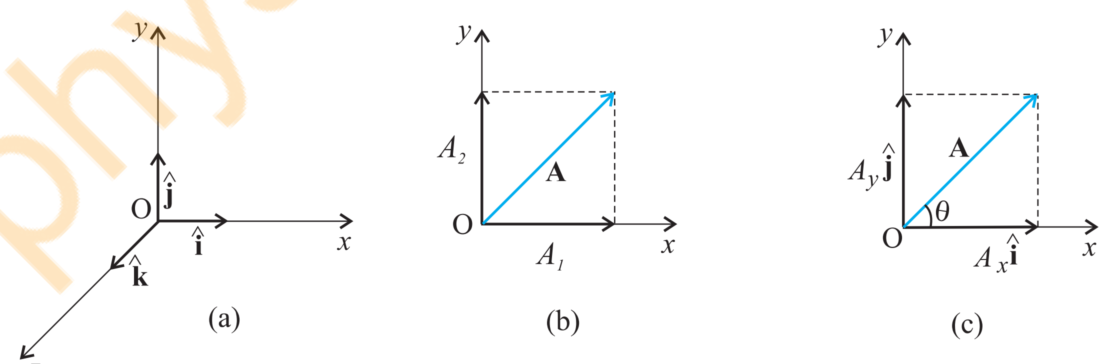

Fig. 4.9 (a) *Unit vectors* $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ *and* $\hat{\mathbf{k}}$ *lie along the x-, y-, and z-axes.* (b) *A vector* **A** *is resolved into its components* A_x *and* A_y *along x-, and y- axes.* (c) $\mathbf{A_1}$ *and* $\mathbf{A_2}$ *expressed in terms of* $\hat{\mathbf{i}}$ *and* $\hat{\mathbf{j}}$.