arranged head to tail, this graphical method is called the **head-to-tail method**. The two vectors and their resultant form three sides of a triangle, so this method is also known as **triangle method of vector addition**. If we find the resultant of **B** + **A** as in Fig. 4.4(c), the same vector **R** is obtained. Thus, vector addition is **commutative**:

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A} \qquad (4.1)$$

The addition of vectors also obeys the associative law as illustrated in Fig. 4.4(d). The result of adding vectors **A** and **B** first and then adding vector **C** is the same as the result of adding **B** and **C** first and then adding vector **A** :

$$(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C}) \qquad (4.2)$$

What is the result of adding two equal and opposite vectors ? Consider two vectors **A** and –**A** shown in Fig. 4.3(b). Their sum is **A** + (–**A**). Since the magnitudes of the two vectors are the same, but the directions are opposite, the resultant vector has zero magnitude and is represented by **0** called a **null vector** or a **zero vector** :

$$\mathbf{A} - \mathbf{A} = \mathbf{0} \qquad |\mathbf{0}| = 0 \qquad (4.3)$$

Def. of zero vector

Since the magnitude of a null vector is zero, its direction cannot be specified.

The null vector also results when we multiply a vector **A** by the number zero. The main properties of **0** are :

$$\mathbf{A} + \mathbf{0} = \mathbf{A}$$
$$\lambda\, \mathbf{0} = \mathbf{0}$$
$$0\, \mathbf{A} = \mathbf{0} \qquad (4.4)$$

What is the physical meaning of a zero vector? Consider the position and displacement vectors in a plane as shown in Fig. 4.1(a). Now suppose that an object which is at P at time *t*, moves to P′ and then comes back to P. Then, what is its displacement? Since the initial and final positions coincide, the displacement is a "null vector".

**Subtraction of vectors** can be defined in terms of addition of vectors. We define the difference of two vectors **A** and **B** as the sum of two vectors **A** and –**B** :

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) \qquad (4.5)$$

It is shown in Fig 4.5. The vector –**B** is added to vector **A** to get $\mathbf{R}_2 = (\mathbf{A} - \mathbf{B})$. The vector $\mathbf{R}_1 = \mathbf{A} + \mathbf{B}$ is also shown in the same figure for comparison. We can also use the **parallelogram method** to find the sum of two vectors. Suppose we have two vectors **A** and **B**. To add these vectors, we bring their tails to a common origin O as shown in Fig. 4.6(a). Then we draw a line from the head of **A** parallel to **B** and another line from the head of **B** parallel to **A** to complete a parallelogram OQSP. Now we join the point of the intersection of these two lines to the origin O. The resultant vector **R** is directed from the common origin O along the diagonal (OS) of the parallelogram [Fig. 4.6(b)]. In Fig.4.6(c), the triangle law is used to obtain the resultant of **A** and **B** and we see that the two methods yield the same result. Thus, the two methods are equivalent.

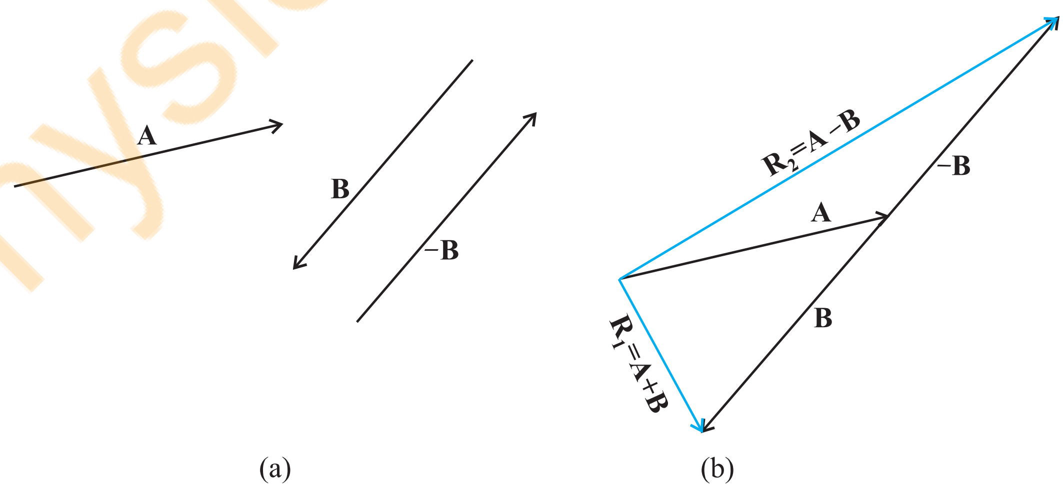


***Fig. 4.5*** (a) *Two vectors* **A** *and* **B**, – **B** *is also shown.* (b) *Subtracting vector* **B** *from vector* **A** *– the result is* $\mathbf{R}_2$. *For comparison, addition of vectors* **A** *and* **B**, *i.e.* $\mathbf{R}_1$ *is also shown.*