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$$\boxed{a_c = \left(\frac{v}{R}\right) v = v^2/R} \quad (4.44)$$

Thus, the acceleration of an object moving with speed v in a circle of radius R has a magnitude v^2/R and is always **directed towards the centre**. This is why this acceleration is called **centripetal acceleration** (a term proposed by Newton). A thorough analysis of centripetal acceleration was first published in 1673 by the Dutch scientist Christiaan Huygens (1629-1695) but it was probably known to Newton also some years earlier. "Centripetal" comes from a Greek term which means 'centre-seeking'. Since v and R are constant, the magnitude of the centripetal acceleration is also constant. However, the direction changes — pointing always towards the centre. Therefore, a centripetal acceleration is not a constant vector.

Imp

We have another way of describing the velocity and the acceleration of an object in uniform circular motion. As the object moves from P to P′ in time Δt (= $t' - t$), the line CP (Fig. 4.19) turns through an angle $\Delta\theta$ as shown in the figure. $\Delta\theta$ is called angular distance. We define the angular speed ω (Greek letter omega) as the time rate of change of angular displacement :

$$\omega = \frac{\Delta\theta}{\Delta t} \quad (4.45)$$

Now, if the distance travelled by the object during the time Δt is Δs, i.e. PP' is Δs, then :

$$v = \frac{\Delta s}{\Delta t}$$

but $\Delta s = R\,\Delta\theta$. Therefore :

$$v = R\frac{\Delta\theta}{\Delta t} = R\,\omega$$

$$\boxed{v = R\,\omega} \quad (4.46)$$

We can express centripetal acceleration a_c in terms of angular speed :

$$a_c = \frac{v^2}{R} = \frac{\omega^2 R^2}{R} = \omega^2 R$$

$$\boxed{a_c = \omega^2 R} \quad (4.47)$$

The time taken by an object to make one revolution is known as its time period T and the number of revolution made in one second is called its frequency ν (=$1/T$). However, during this time the distance moved by the object is $s = 2\pi R$.

Therefore, $v = 2\pi R/T = 2\pi R\nu$ (4.48)

In terms of frequency ν, we have

$\omega = 2\pi\nu$

$v = 2\pi R\nu$

$a_c = 4\pi^2 \nu^2 R$ (4.49)

These formulae are valid only for uniform circular motion

Example 4.10 An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s. (a) What is the angular speed, and the linear speed of the motion? (b) Is the acceleration vector a constant vector ? What is its magnitude ?

Answer This is an example of uniform circular motion. Here R = 12 cm. The angular speed ω is given by

$\omega = 2\pi/T = 2\pi \times 7/100 = 0.44$ rad/s

The linear speed v is :

$v = \omega R = 0.44\ \text{s}^{-1} \times 12\ \text{cm} = 5.3\ \text{cm s}^{-1}$

The direction of velocity $\boldsymbol{v}$ is along the tangent to the circle at every point. The acceleration is directed towards the centre of the circle. Since this direction changes continuously, acceleration here is *not* a constant vector. However, the magnitude of acceleration is constant:

$a = \omega^2 R = (0.44\ \text{s}^{-1})^2\ (12\ \text{cm})$

$= 2.3\ \text{cm s}^{-2}$

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