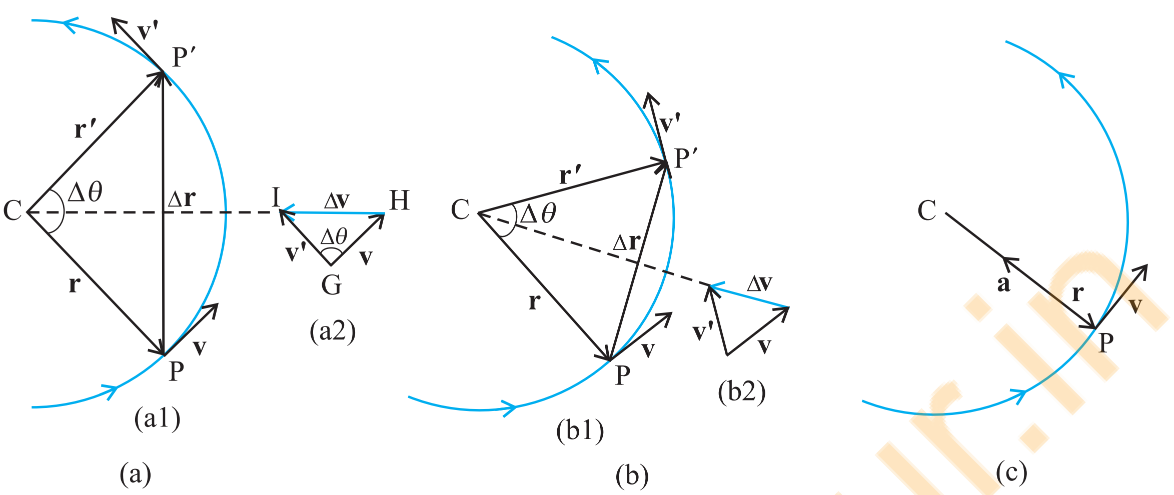

Fig. 4.19 *Velocity and acceleration of an object in uniform circular motion. The time interval Δt decreases from (a) to (c) where it is zero. The acceleration is directed, at each point of the path, towards the centre of the circle.*

Let **r** and **r**′ be the position vectors and **v** and **v**′ the velocities of the object when it is at point *P* and *P*′ as shown in Fig. 4.19(a). By definition, velocity at a point is along the tangent at that point in the direction of motion. The velocity vectors **v** and **v**′ are as shown in Fig. 4.19(a1). $\Delta\mathbf{v}$ is obtained in Fig. 4.19 (a2) using the triangle law of vector addition. Since the path is circular, **v** is perpendicular to **r** and so is **v**′ to **r**′. Therefore, $\Delta\mathbf{v}$ is perpendicular to $\Delta\mathbf{r}$. Since average acceleration is along $\Delta\mathbf{v}$ $\left(\bar{\mathbf{a}} = \frac{\Delta\mathbf{v}}{\Delta t}\right)$, the average acceleration $\bar{\mathbf{a}}$ is perpendicular to $\Delta\mathbf{r}$. If we place $\Delta\mathbf{v}$ on the line that bisects the angle between **r** and **r**′, we see that it is directed towards the centre of the circle. Figure 4.19(b) shows the same quantities for smaller time interval. $\Delta\mathbf{v}$ and hence $\bar{\mathbf{a}}$ is again directed towards the centre. In Fig. 4.19(c), $\Delta t \to 0$ and the average acceleration becomes the instantaneous acceleration. It is directed towards the centre*. Thus, we find that the acceleration of an object in uniform circular motion is always directed towards the centre of the circle. Let us now find the magnitude of the acceleration.

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The magnitude of **a** is, by definition, given by

$$|\mathbf{a}| = \lim_{\Delta t \to 0} \frac{|\Delta\mathbf{v}|}{\Delta t}$$

Let the angle between position vectors **r** and **r**′ be $\Delta\theta$. Since the velocity vectors **v** and **v**′ are always perpendicular to the position vectors, the angle between them is also $\Delta\theta$. Therefore, the triangle CPP′ formed by the position vectors and the triangle GHI formed by the velocity vectors **v**, **v**′ and $\Delta\mathbf{v}$ are similar (Fig. 4.19a). Therefore, the ratio of the base-length to side-length for one of the triangles is equal to that of the other triangle. That is :

$$\frac{|\Delta\mathbf{v}|}{v} = \frac{|\Delta\mathbf{r}|}{R}$$

Or, $$|\Delta\mathbf{v}| = v\frac{|\Delta\mathbf{r}|}{R}$$

Therefore,

$$|\mathbf{a}| = \lim_{\Delta t \to 0} \frac{|\Delta\mathbf{v}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{v|\Delta\mathbf{r}|}{R\Delta t} = \frac{v}{R}\lim_{\Delta t \to 0} \frac{|\Delta\mathbf{r}|}{\Delta t}$$

If Δt is small, $\Delta\theta$ will also be small and then arc *PP*′ can be approximately taken to be $|\Delta\mathbf{r}|$:

$$|\Delta\mathbf{r}| \cong v\Delta t$$

$$\frac{|\Delta\mathbf{r}|}{\Delta t} \cong v$$

Or, $$\lim_{\Delta t \to 0} \frac{|\Delta\mathbf{r}|}{\Delta t} = v$$

Therefore, the centripetal acceleration a_c is :

* *In the limit $\Delta t \to 0$, $\Delta\mathbf{r}$ becomes perpendicular to* **r**. *In this limit $\Delta\mathbf{v} \to \mathbf{0}$ and is consequently also perpendicular to* **V**. *Therefore, the acceleration is directed towards the centre, at each point of the circular path.*

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