Answer We choose the origin of the x-,and y-axis at the edge of the cliff and t = 0 s at the instant the stone is thrown. Choose the positive direction of x-axis to be along the initial velocity and the positive direction of y-axis to be the vertically upward direction. The x-, and y-components of the motion can be treated independently. The equations of motion are :

$$x(t) = x_o + v_{ox}\, t$$

$$y(t) = y_o + v_{oy}\, t + (1/2)\, a_y\, t^2$$

Here, $x_o = y_o = 0$, $v_{oy} = 0$, $a_y = -g = -9.8$ m s^{-2},
$v_{ox} = 15$ m s^{-1}.

The stone hits the ground when $y(t) = -490$ m.

$$-490 \text{ m} = -(1/2)(9.8)\, t^2.$$

This gives $t = 10$ s.

The velocity components are $v_x = v_{ox}$ and $v_y = v_{oy} - g\,t$

so that when the stone hits the ground :

$$v_{ox} = 15 \text{ m s}^{-1}$$

$$v_{oy} = 0 - 9.8 \times 10 = -98 \text{ m s}^{-1}$$

Therefore, the speed of the stone is

$$\sqrt{v_x^2 + v_y^2} = \sqrt{15^2 + 98^2} = 99 \text{ m s}^{-1}$$

◀

Example 4.9 A cricket ball is thrown at a speed of 28 m s^{-1} in a direction 30° above the horizontal. Calculate (a) the maximum height, (b) the time taken by the ball to return to the same level, and (c) the distance from the thrower to the point where the ball returns to the same level.

Answer (a) The maximum height is given by

$$h_m = \frac{(v_0 \sin\theta_o)^2}{2g} = \frac{(28 \sin 30°)^2}{2\ (9.8)} \text{ m}$$

$$= \frac{14 \times 14}{2 \times 9.8} = 10.0 \text{ m}$$

(b) The time taken to return to the same level is

$$T_f = (2\ v_o \sin\ \theta_o)/g = (2 \times 28 \times \sin 30°)/9.8$$

$$= 28/9.8 \text{ s} = 2.9 \text{ s}$$

(c) The distance from the thrower to the point where the ball returns to the same level is

$$R = \frac{(v_o^2 \sin 2\theta_o)}{g} = \frac{28 \times 28 \times \sin 60^o}{9.8} = 69 \text{ m}$$

◀

Neglecting air resistance - what does the assumption really mean?

While treating the topic of projectile motion, we have stated that we assume that the air resistance has no effect on the motion of the projectile. You must understand what the statement really means. Friction, force due to viscosity, air resistance are all dissipative forces. In the presence of any of such forces opposing motion, any object will lose some part of its initial energy and consequently, momentum too. Thus, a projectile that traverses a parabolic path would certainly show deviation from its idealised trajectory in the presence of air resistance. It will not hit the ground with the same speed with which it was projected from it. In the absence of air resistance, the x-component of the velocity remains constant and it is only the y-component that undergoes a continuous change. However, in the presence of air resistance, both of these would get affected. That would mean that the range would be less than the one given by Eq. (4.43). Maximum height attained would also be less than that predicted by Eq. (4.42). Can you then, anticipate the change in the time of flight?

In order to avoid air resistance, we will have to perform the experiment in vacuum or under low pressure, which is not easy. When we use a phrase like 'neglect air resistance', we imply that the change in parameters such as range, height etc. is much smaller than their values without air resistance. The calculation without air resistance is much simpler than that with air resistance.

4.11 UNIFORM CIRCULAR MOTION

When an object follows a circular path at a constant speed, the motion of the object is called **uniform circular motion**. The word "uniform" refers to the speed, which is uniform (constant) throughout the motion. Suppose an object is moving with uniform speed v in a circle of radius R as shown in Fig. 4.19. Since the velocity of the object is changing continuously in direction, the object undergoes acceleration. Let us find the magnitude and the direction of this acceleration.