$$y = (\tan\theta_o)\,x - \frac{g}{2\,(v_o\cos\theta_o)^2}\,x^2 \qquad (4.40)$$

Now, since $g$, $\theta_o$ and $v_o$ are constants, Eq. (4.40) is of the form $y = a\,x + b\,x^2$, in which $a$ and $b$ are constants. This is the equation of a parabola, i.e. the path of the projectile is a parabola (Fig. 4.18).

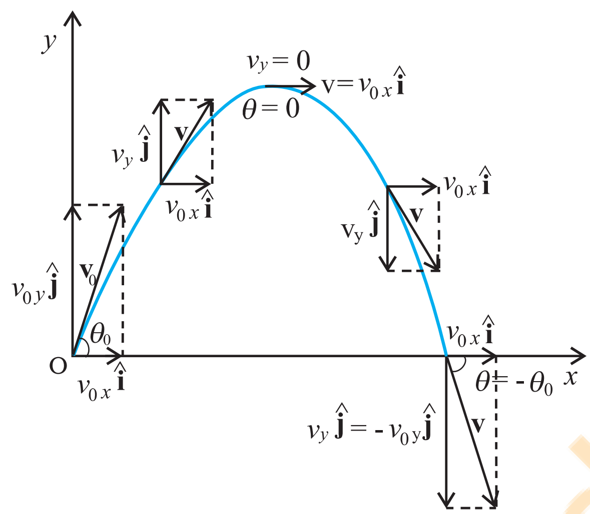


*Fig. 4.18 The path of a projectile is a parabola.*

### Time of maximum height

$t_m = ?$

How much time does the projectile take to reach the maximum height ? Let this time be denoted by $t_m$. Since at this point, $v_y = 0$, we have from Eq. (4.39):

$$v_y = v_o\sin\theta_o - g\,t_m = 0$$

Or, $\quad t_m = v_o\sin\theta_o/g \qquad (4.41a)$

The total time $T_f$ during which the projectile is in flight can be obtained by putting $y = 0$ in Eq. (4.38). We get :

$T_f = \frac{2u_y}{g}$

$$T_f = 2\,(v_o\sin\theta_o)/g \qquad (4.41b)$$

$T_f$ is known as the **time of flight** of the projectile. We note that $T_f = 2\,t_m$, which is expected because of the symmetry of the parabolic path.

### Maximum height of a projectile

The maximum height $h_m$ reached by the projectile can be calculated by substituting $t = t_m$ in Eq. (4.38) :

$$y = h_m = (v_0\sin\theta_0)\left(\frac{v_0\sin\theta_0}{g}\right) - \frac{g}{2}\left(\frac{v_0\sin\theta_0}{g}\right)^2$$

Use formula $v_y^2 = u_y^2 + 2a_yS_y$

$v_y = 0,\ u_y = v_0\sin\theta_0,\ a_y = -g,\ S_y = h_m$

Or, $\quad h_m = \dfrac{(v_0\sin\theta_0)^2}{2g} \qquad (4.42)$

$h_m = \frac{u_y^2}{2g}$

### Horizontal range of a projectile

The horizontal distance travelled by a projectile from its initial position ($x = y = 0$) to the position where it passes $y = 0$ during its fall is called the **horizontal range**, $R$. It is the distance travelled during the time of flight $T_f$ . Therefore, the range $R$ is

$$R = (v_o\cos\theta_o)\,(T_f)$$

$$= (v_o\cos\theta_o)\ (2\,v_o\sin\theta_o)/g$$

$R = \frac{2u_xu_y}{g}$

Or, $\quad R = \dfrac{v_0^2\sin 2\theta_0}{g} \qquad (4.43a)$

Equation (4.43a) shows that for a given projection velocity $v_o$ , $R$ is maximum when $\sin 2\theta_0$ is maximum, i.e., when $\theta_0 = 45^0$.

The maximum horizontal range is, therefore,

$$R_m = \frac{v_0^2}{g} \qquad (4.43b)$$

**Example 4.7** Galileo, in his book **Two new sciences,** stated that "for elevations which exceed or fall short of 45° by equal amounts, the ranges are equal". Prove this statement.

***Answer*** For a projectile launched with velocity $\mathbf{v}_o$ at an angle $\theta_o$ , the range is given by

$$R = \frac{v_0^2\sin 2\theta_0}{g}$$

Now, for angles, $(45° + \alpha)$ and $(45° - \alpha)$, $2\theta_o$ is $(90° + 2\alpha)$ and $(90° - 2\alpha)$ , respectively. The values of $\sin(90° + 2\alpha)$ and $\sin(90° - 2\alpha)$ are the same, equal to that of $\cos 2\alpha$. Therefore, ranges are equal for elevations which exceed or fall short of 45° by equal amounts $\alpha$.

**Example 4.8** A hiker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an initial speed of 15 m s$^{-1}$. Neglecting air resistance, find the time taken by the stone to reach the ground, and the speed with which it hits the ground. (Take $g$ = 9.8 m s$^{-2}$).

horizontal Projectile