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This relative velocity vector as shown in Fig. 4.16 makes an angle θ with the vertical. It is given by

$$\tan \theta = \frac{v_b}{v_r} = \frac{12}{35} = 0.343$$

 $\theta \cong 19^\circ$

Or,

Therefore, the woman should hold her umbrella at an angle of about 19° with the vertical towards the west.

Note carefully the difference between this Example and the Example 4.1. In Example 4.1, the boy experiences the resultant (vector sum) of two velocities while in this example, the woman experiences the velocity of rain relative to the bicycle (the vector difference of the two velocities).

4.10 PROJECTILE MOTION

As an application of the ideas developed in the previous sections, we consider the motion of a projectile. An object that is in flight after being thrown or projected is called a **projectile**. Such a projectile might be a football, a cricket ball, a baseball or any other object. The motion of a projectile may be thought of as the result of two separate, simultaneously occurring components of motions. One component is along a horizontal direction without any acceleration and the other along the vertical direction with constant acceleration due to the force of gravity. It was Galileo who first stated this independency of the horizontal and the vertical components of projectile motion in his Dialogue on the great world systems (1632).

In our discussion, we shall assume that the air resistance has negligible effect on the motion of the projectile. Suppose that the projectile is launched with velocity \mathbf{v}_{o} that makes an angle θ_{o} with the *x*-axis as shown in Fig. 4.17.

After the object has been projected, the acceleration acting on it is that due to gravity which is directed vertically downward:

$$\mathbf{a} = -g \mathbf{j}$$

Or, $a_x = 0$, $a_y = -g$ (4.36) The components of initial velocity **v** are :

$$v_{or} = v_o \cos \theta_o$$

$$v_{oy} = v_o \sin \theta_o \tag{4.37}$$



Fig 4.17 Motion of an object projected with velocity \boldsymbol{v}_{o} at angle θ_{o} .

If we take the initial position to be the origin of the reference frame as shown in Fig. 4.17, we have :

$$x = 0, y = 0$$

Then, Eq.(4.34b) becomes :

$$x = v_{ox} t = (v_o \cos \theta_o) t$$

and
$$y = (v_o \sin \theta_o) t - (\frac{1}{2})g t^2$$
(4.38)

The components of velocity at time t can be obtained using Eq.(4.33b) :

 $v_{x} = v_{ox} = v_{o} \cos \theta_{o} \longrightarrow \mathcal{V}_{X} = \mathcal{U}_{X} \left(\stackrel{\circ \circ}{\cdot} \mathcal{Q}_{X} = 0 \right)$

 $v_y = v_o \sin \theta_o - gt \rightarrow \psi_y = u_y (4^{39}) a_y t$ ($\alpha_y = -g$) Equation (4.38) gives the x-, and y-coordinates of the position of a projectile at time t in terms of two parameters — initial speed v_o and projection angle θ_o . Notice that the choice of mutually perpendicular x-, and y-directions for the analysis of the projectile motion has resulted in a simplification. One of the components of velocity, i.e. x-component remains constant throughout the motion and only the y- component changes, like an object in free fall in vertical direction. This is shown graphically at few instants in Fig. 4.18. Note that at the point of maximum height, $v_y = 0$ and therefore,

$$\theta = \tan^{-1} \frac{v_y}{v_x} = 0$$
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Equation of path of a projectile

What is the shape of the path followed by the projectile? This can be seen by eliminating the time between the expressions for x and y as given in Eq. (4.38). We obtain:

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