$$= \mathbf{v_0}t + \frac{1}{2}\mathbf{a}t^2$$

$\vec{r}-\vec{r}_0=\vec{v}_0 t+\frac{1}{2}\vec{a}t^2$

Or, $\mathbf{r} = \mathbf{r_0} + \mathbf{v_0}t + \frac{1}{2}\mathbf{a}t^2$ (4.34a)

$\vec{s}=\vec{u}t+\frac{1}{2}\vec{a}t^2$

It can be easily verified that the derivative of Eq. (4.34a), i.e. $\frac{d\mathbf{r}}{dt}$ gives Eq.(4.33a) and it also satisfies the condition that at t=0, $\mathbf{r} = \mathbf{r_o}$. Equation (4.34a) can be written in component form as

$x-x_0=u_x t+\frac{1}{2}a_x t^2$

$s_x=u_x t+\frac{1}{2}a_x t^2$

$$x = x_0 + v_{ox}t + \frac{1}{2}a_x t^2$$

$s_y=u_y t+\frac{1}{2}a_y t^2$

$$y = y_0 + v_{oy}t + \frac{1}{2}a_y t^2 \quad (4.34b)$$

One immediate interpretation of Eq.(4.34b) is that the motions in x- and y-directions can be treated independently of each other. That is, **motion in a plane (two-dimensions) can be treated as two separate simultaneous one-dimensional motions with constant acceleration along two perpendicular directions.** This is an important result and is useful in analysing motion of objects in two dimensions. A similar result holds for three dimensions. The choice of perpendicular directions is convenient in many physical situations, as we shall see in section 4.10 for projectile motion.

Imp:

▶ ***Example 4.5*** A particle starts from origin at t = 0 with a velocity 5.0 $\hat{\mathbf{i}}$ m/s and moves in x-y plane under action of a force which produces a constant acceleration of $(3.0\hat{\mathbf{i}}+2.0\hat{\mathbf{j}})$ m/s². (a) What is the y-coordinate of the particle at the instant its x-coordinate is 84 m ? (b) What is the speed of the particle at this time ?

Answer From Eq. (4.34a) for $\mathbf{r_0}$ = 0, the position of the particle is given by

$$\mathbf{r}(t) = \mathbf{v_0}t + \frac{1}{2}\mathbf{a}t^2$$

$$= 5.0\hat{\mathbf{i}}\,t + (1/2)(3.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})\,t^2$$

$$= (5.0\,t + 1.5\,t^2)\hat{\mathbf{i}} + 1.0\,t^2\hat{\mathbf{j}}$$

Therefore, $x(t) = 5.0\,t + 1.5\,t^2$

$$y(t) = +1.0\,t^2$$

Given $x(t)$ = 84 m, t = ?

$$5.0\,t + 1.5\,t^2 = 84 \Rightarrow t = 6 \text{ s}$$

At t = 6 s, y = 1.0 (6)² = 36.0 m

Now, the velocity $\mathbf{v} = \frac{d\mathbf{r}}{dt} = (5.0 + 3.0\,t)\hat{\mathbf{i}} + 2.0\,t\,\hat{\mathbf{j}}$

At t = 6 s, $\mathbf{v} = 23.0\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}}$

speed $= |\mathbf{v}| = \sqrt{23^2 + 12^2} \cong 26 \text{ m s}^{-1}$. ◀

4.9 RELATIVE VELOCITY IN TWO DIMENSIONS

The concept of relative velocity, introduced in section 3.7 for motion along a straight line, can be easily extended to include motion in a plane or in three dimensions. Suppose that two objects A and B are moving with velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ (each with respect to some common frame of reference, say ground.). Then, velocity of object **A relative to that of B is** :

$$\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B \quad (4.35a)$$

and similarly, the velocity of object B *relative to that of* A is :

$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A$$

$\vec{v}_{rel}=\vec{v}_{body}-\vec{v}_{obs}$

Therefore, $\mathbf{v}_{AB} = -\mathbf{v}_{BA}$ (4.35b)

and, $|\mathbf{v}_{AB}| = |\mathbf{v}_{BA}|$ (4.35c)

▶ ***Example 4.6*** Rain is falling vertically with a speed of 35 m s⁻¹. A woman rides a bicycle with a speed of 12 m s⁻¹ in east to west direction. What is the direction in which she should hold her umbrella ?

Answer In Fig. 4.16 $\mathbf{v}_r$ represents the velocity of rain and $\mathbf{v}_b$, the velocity of the bicycle, the woman is riding. Both these velocities are with respect to the ground. Since the woman is riding a bicycle, the velocity of rain as experienced by

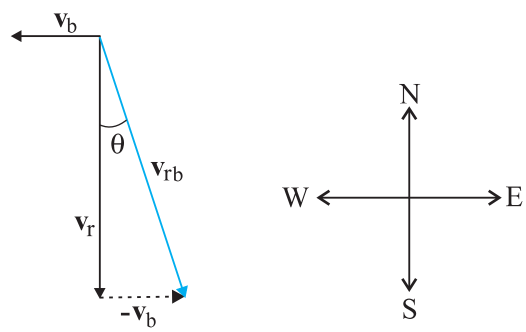

Fig. 4.16

her is the velocity of rain relative to the velocity of the bicycle she is riding. That is $\mathbf{v}_{rb} = \mathbf{v}_r - \mathbf{v}_b$