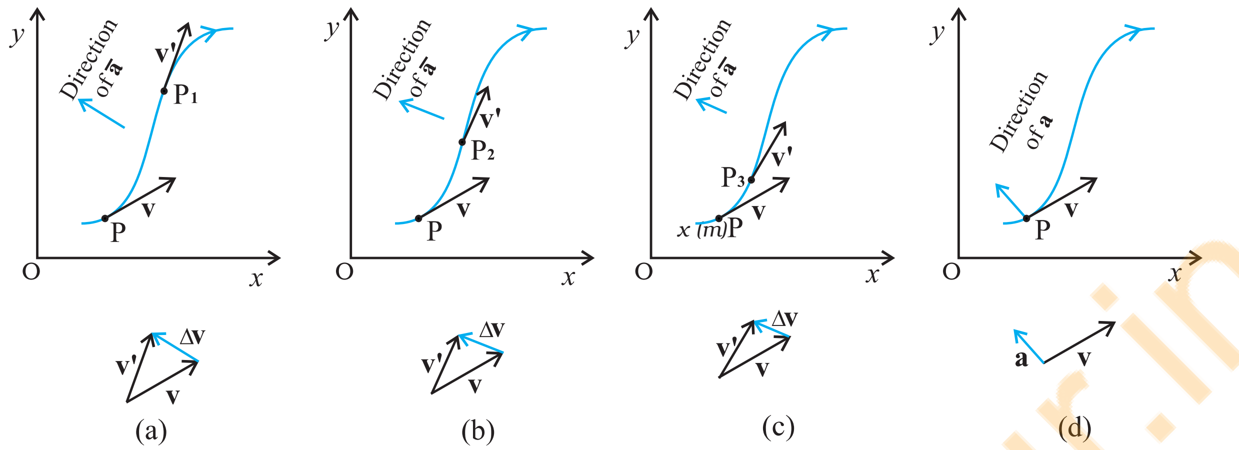

Fig. 4.15 *The average acceleration for three time intervals* (a) Δt_1, (b) Δt_2, *and* (c) Δt_3, $(\Delta t_1 > \Delta t_2 > \Delta t_3)$. (d) *In the limit* $\Delta t \to 0$, *the average acceleration becomes the acceleration.*

Note that in one dimension, the velocity and the acceleration of an object are always along the same straight line (either in the same direction or in the opposite direction). However, for motion in two or three dimensions, velocity and acceleration vectors may have any angle between 0° and 180° between them.

Example 4.4 The position of a particle is given by

$$\mathbf{r} = 3.0t\,\hat{\mathbf{i}} + 2.0t^2\,\hat{\mathbf{j}} + 5.0\,\hat{\mathbf{k}}$$

where t is in seconds and the coefficients have the proper units for $\mathbf{r}$ to be in metres. (a) Find $\mathbf{v}(t)$ and $\mathbf{a}(t)$ of the particle. (b) Find the magnitude and direction of $\mathbf{v}(t)$ at $t = 1.0$ s.

Answer

$$\mathbf{v}(t) = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}\left(3.0\,t\,\hat{\mathbf{i}} + 2.0t^2\,\hat{\mathbf{j}} + 5.0\,\hat{\mathbf{k}}\right)$$

$$= 3.0\hat{\mathbf{i}} + 4.0t\,\hat{\mathbf{j}}$$

$$\mathbf{a}(t) = \frac{d\mathbf{v}}{dt} = +4.0\hat{\mathbf{j}}$$

$a = 4.0$ m s^{-2} along y- direction

At $t = 1.0$ s, $\mathbf{v} = 3.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}}$

It's magnitude is $v = \sqrt{3^2 + 4^2} = 5.0$ m s^{-1}

and direction is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{4}{3}\right) \cong 53^\circ \text{ with } x\text{-axis.}$$

4.8 MOTION IN A PLANE WITH CONSTANT ACCELERATION

Suppose that an object is moving in x-y plane and its acceleration $\mathbf{a}$ is constant. Over an interval of time, the average acceleration will equal this constant value. Now, let the velocity of the object be $\mathbf{v}_0$ at time $t = 0$ and $\mathbf{v}$ at time t.

Then, by definition

$$\mathbf{a} = \frac{\mathbf{v} - \mathbf{v_0}}{t - 0} = \frac{\mathbf{v} - \mathbf{v_0}}{t}$$

Or, $\mathbf{v} = \mathbf{v_0} + \mathbf{a}t$ $\rightarrow \vec{v} = \vec{u} + \vec{a}t$ (4.33a)

In terms of components :

$v_x = v_{ox} + a_x t$ $\rightarrow v_x = u_x + a_x t$

$v_y = v_{oy} + a_y t$ $\rightarrow v_y = u_y + a_y t$ (4.33b)

Let us now find how the position $\mathbf{r}$ changes with time. We follow the method used in the one-dimensional case. Let $\mathbf{r_o}$ and $\mathbf{r}$ be the position vectors of the particle at time 0 and t and let the velocities at these instants be $\mathbf{v_o}$ and $\mathbf{v}$. Then, over this time interval t, the average velocity is $(\mathbf{v_o} + \mathbf{v})/2$. The displacement is the average velocity multiplied by the time interval : because $\vec{a} = \text{const.}$

$$\mathbf{r} - \mathbf{r_0} = \left(\frac{\mathbf{v} + \mathbf{v_0}}{2}\right)t = \left(\frac{(\mathbf{v_0} + \mathbf{a}t) + \mathbf{v_0}}{2}\right)t$$

$$\vec{S} = \vec{v}_{av} \cdot t = \left(\frac{\vec{v} + \vec{u}}{2}\right)t$$