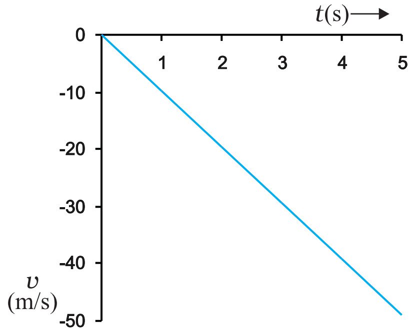
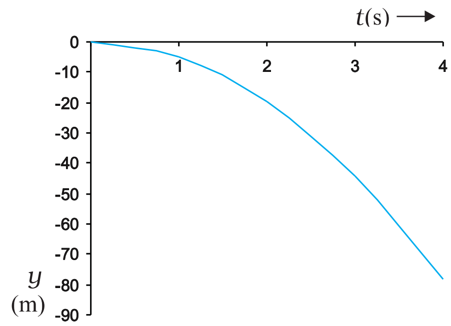


(b)



(c)

Fig. 3.14 *Motion of an object under free fall.*
(a) Variation of acceleration with time.
(b) Variation of velocity with time.
(c) Variation of distance with time

Imp

Example 3.6 Galileo's law of odd numbers : *"The distances traversed, during equal intervals of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity [namely, 1: 3: 5: 7......]."* Prove it.

Answer Let us divide the time interval of motion of an object under free fall into many equal intervals τ and find out the distances traversed during successive intervals of time. Since initial velocity is zero, we have

$$\boxed{y = -\frac{1}{2} g t^2}$$

Using this equation, we can calculate the position of the object after different time intervals, 0, τ, 2τ, 3τ... which are given in second column of Table 3.2. If we take $(-1/2)\, g\tau^2$ as y_0 — the position coordinate after first time interval τ, then third column gives the positions in the unit of y_o. The fourth column gives the distances traversed in successive τs. We find that the distances are in the simple ratio 1: 3: 5: 7: 9: 11... as shown in the last column. This law was established by Galileo Galilei (1564-1642) who was the first to make quantitative studies of free fall.

Example 3.7 Stopping distance of vehicles : When brakes are applied to a moving vehicle, the distance it travels before stopping is called stopping distance. It is an important factor for road safety and depends on the initial velocity (v_0) and the braking capacity, or deceleration, $-a$ that is caused by the braking. Derive an expression for stopping distance of a vehicle in terms of v_o and a.

Answer Let the distance travelled by the vehicle before it stops be d_s. Then, using equation of motion $v^2 = v_o^2 + 2\, ax$, and noting that $v = 0$, we have the stopping distance

$$\boxed{d_s = \frac{-v_0^2}{2a}}$$

Imp

Thus, the stopping distance is proportional to the square of the initial velocity. Doubling the

Table 3.2

t	y	y in terms of y_o [=(- ½) g τ^2]	Distance traversed in successive intervals	Ratio of distances traversed
0	0	0		
τ	$-(1/2)\, g\, \tau^2$	y_o	y_o	1
$2\,\tau$	$-4(1/2)\, g\, \tau^2$	$4\, y_o$	$3\, y_o$	3
$3\,\tau$	$-9(1/2)\, g\, \tau^2$	$9\, y_o$	$5\, y_o$	5
$4\,\tau$	$-16(1/2)\, g\, \tau^2$	$16\, y_o$	$7\, y_o$	7
$5\,\tau$	$-25(1/2)\, g\, \tau^2$	$25\, y_o$	$9\, y_o$	9
$6\,\tau$	$-36(1/2)\, g\, \tau^2$	$36\, y_o$	$11\, y_o$	11