This equation can also be obtained by substituting the value of t from Eq. (3.6) into Eq. (3.8). Thus, we have obtained three important equations :

$$v = v_0 + at$$

$$x = v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2ax \qquad (3.11a)$$

connecting five quantities v_0, v, a, t and x. These are kinematic equations of rectilinear motion for constant acceleration.

The set of Eq. (3.11a) were obtained by assuming that at t = 0, the position of the particle, x is 0. We can obtain a more general equation if we take the position coordinate at t = 0 as non-zero, say x_0. Then Eqs. (3.11a) are modified (replacing x by $x - x_0$) to :

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \qquad (3.11b)$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad (3.11c)$$

Example 3.3 Obtain equations of motion for constant acceleration using method of calculus.

Answer By definition

$$a = \frac{dv}{dt}$$

$$dv = a\,dt$$

Integrating both sides

$$\int_{v_0}^{v} dv = \int_0^t a \; dt$$

$$= a\int_0^t dt \qquad (a \text{ is constant})$$

$$v - v_0 = at$$

$$v = v_0 + at$$

Further, $$v = \frac{dx}{dt}$$

$$dx = v\,dt$$

Integrating both sides

$$\int_{x_0}^{x} dx = \int_0^t v \; dt$$

$$= \int_0^t (v_0 + at) \; dt$$

$$x - x_0 = v_0 \; t + \frac{1}{2}a \; t^2$$

$$x \; = x_0 + v_0 \; t + \frac{1}{2} \; a \; t^2$$

We can write

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

or, $v \, dv = a \, dx$

Integrating both sides,

$$\int_{v_0}^{v} v \, dv \; = \; \int_{x_0}^{x} a \, dx$$

$$\frac{v^2 - v_0^2}{2} \; = \; a\,(x - x_0)$$

$$v^2 = v_0^2 + 2a\,(x - x_0)$$

The advantage of this method is that it can be used for motion with non-uniform acceleration also.

Now, we shall use these equations to some important cases.

Example 3.4 A ball is thrown vertically upwards with a velocity of 20 m s^{-1} from the top of a multistorey building. The height of the point from where the ball is thrown is 25.0 m from the ground. (a) How high will the ball rise ? and (b) how long will it be before the ball hits the ground? Take g = 10 m s^{-2}.

Answer (a) Let us take the y-axis in the vertically upward direction with zero at the ground, as shown in Fig. 3.13.

Now v_o = + 20 m s^{-1},

a = $-g$ = –10 m s^{-2},

v = 0 m s^{-1}

If the ball rises to height y from the point of launch, then using the equation

$$v^2 = \; v_0^2 + 2 \; a \; \left(y - y_0\right)$$

we get

$$0 = (20)^2 + 2(-10)(y - y_0)$$

Solving, we get, $(y - y_0)$ = 20 m.

(b) We can solve this part of the problem in two ways. **Note carefully the methods used**.