statement requires use of calculus. We can, however, see that it is true for the simple case of an object moving with constant velocity u. Its velocity-time graph is as shown in Fig. 3.11.

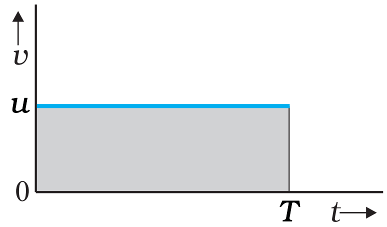

Fig. 3.11 Area under υ–t curve equals displacement of the object over a given time interval.

The v-t curve is a straight line parallel to the time axis and the area under it between $t = 0$ and $t = T$ is the area of the rectangle of height u and base T. Therefore, area $= u \times T = uT$ which is the displacement in this time interval. How come in this case an area is equal to a distance? Think! Note the dimensions of quantities on the two coordinate axes, and you will arrive at the answer.

Note that the x-t, v-t, and a-t graphs shown in several figures in this chapter have sharp kinks at some points implying that the functions are not differentiable at these points. In any realistic situation, the functions will be differentiable at all points and the graphs will be smooth.

What this means physically is that acceleration and velocity cannot change values abruptly at an instant. Changes are always continuous.

3.6 KINEMATIC EQUATIONS FOR UNIFORMLY ACCELERATED MOTION

For uniformly accelerated motion, we can derive some simple equations that relate displacement (x), time taken (t), initial velocity (v_0), final velocity (v) and acceleration (a). Equation (3.6) already obtained gives a relation between final and initial velocities v and v_0 of an object moving with uniform acceleration a :

$v=u+at$

$$v = v_0 + at \qquad (3.6)$$

This relation is graphically represented in Fig. 3.12. The area under this curve is :

Area between instants 0 and t = Area of triangle ABC + Area of rectangle OACD

$$= \frac{1}{2}(v-v_0)t + v_0 t$$

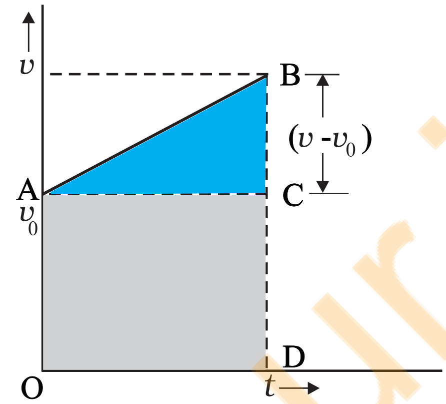

Fig. 3.12 Area under v-t curve for an object with uniform acceleration.

As explained in the previous section, the area under v-t curve represents the displacement. Therefore, the displacement x of the object is :

$$x = \frac{1}{2}(v-v_0)t + v_0 t \qquad (3.7)$$

But $\quad v - v_0 = a\,t$

Therefore, $x = \frac{1}{2} a\, t^2 + v_0 t$

or, $$x = v_0 t + \frac{1}{2} a t^2 \qquad (3.8)$$

$S=ut+\frac{1}{2}at^2$

Equation (3.7) can also be written as

$$x = \frac{v + v_0}{2} t = \bar{v}\, t \qquad (3.9a)$$

where,

For $a=$ const

$v_{av} = \frac{u+v}{2}$

$$\bar{v} = \frac{v + v_0}{2} \quad \textbf{(constant acceleration only)} \qquad (3.9b)$$

Equations (3.9a) and (3.9b) mean that the object has undergone displacement x with an average velocity equal to the arithmetic average of the initial and final velocities.

From Eq. (3.6), $t = (v - v_0)/a$. Substituting this in Eq. (3.9a), we get

$$x = \bar{v}\, t = \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right) = \frac{v^2 - v_0^2}{2a}$$

$$v^2 = v_0^2 + 2ax \qquad (3.10)$$

 $v^2=u^2+2as$