Example 3.2 *The position of an object moving along x-axis is given by $x = a + bt^2$ where $a = 8.5$ m, $b = 2.5$ m s^{-2} and t is measured in seconds. What is its velocity at $t = 0$ s and $t = 2.0$ s. What is the average velocity between $t = 2.0$ s and $t = 4.0$ s ?*

Answer In notation of differential calculus, the velocity is

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(a + bt^2\right) = 2b\,t = 5.0\ t \text{ m s}^{-1}$$

At $t = 0$ s, $v = 0$ m s^{-1} and at $t = 2.0$ s, $v = 10$ m s^{-1} .

$$\text{Average velocity} = \frac{x(4.0) - x(2.0)}{4.0 - 2.0}$$

$$= \frac{a + 16b - a - 4b}{2.0} = 6.0 \times b$$

$$= 6.0 \times 2.5 = 15 \text{ m s}^{-1}$$

From Fig. 3.7, we note that during the period t =10 s to 18 s the velocity is constant. Between period t =18 s to t = 20 s, it is uniformly decreasing and during the period t = 0 s to t = 10 s, it is increasing. **Note that for uniform motion, velocity is the same as the average velocity at all instants**.

Instantaneous speed or simply speed is the magnitude of velocity. For example, a velocity of + 24.0 m s^{-1} and a velocity of – 24.0 m s^{-1} — both have an associated speed of 24.0 m s^{-1}. It should be noted that though average speed over a finite interval of time is greater or equal to the magnitude of the average velocity, instantaneous speed at an instant is equal to the magnitude of the instantaneous velocity at that instant. Why so ?

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3.5 ACCELERATION

The velocity of an object, in general, changes during its course of motion. How to describe this change? Should it be described as the rate of change in velocity **with distance** or **with time** ? This was a problem even in Galileo's time. It was first thought that this change could be described by the rate of change of velocity with distance. But, through his studies of motion of freely falling objects and motion of objects on an inclined plane, Galileo concluded that the rate of change of velocity with time is a constant of motion for all objects in free fall. On the other hand, the change in velocity with distance is not constant – it decreases with the increasing distance of fall.

This led to the concept of acceleration as the rate of change of velocity with time. Def.

The average acceleration $\overline{a}$ over a time interval is defined as the change of velocity divided by the time interval : Def. of av. acceleration

$$\overline{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \quad (3.4)$$

where v_2 and v_1 are the instantaneous velocities or simply velocities at time t_2 and t_1 . It is the average change of velocity per unit time. The SI unit of acceleration is m s^{-2} .

On a plot of velocity versus time, the average acceleration is the slope of the straight line connecting the points corresponding to (v_2, t_2) and (v_1, t_1). The average acceleration for velocity-time graph shown in Fig. 3.7 for different time intervals 0 s - 10 s, 10 s – 18 s, and 18 s – 20 s are :

0 s - 10 s $\quad \overline{a} = \dfrac{(24 - 0)\text{ m s}^{-1}}{(10 - 0)\text{ s}} = 2.4 \text{ m s}^{-2}$

10 s - 18 s $\quad \overline{a} = \dfrac{(24 - 24)\text{ m s}^{-1}}{(18 - 10)\text{ s}} = 0 \text{ m s}^{-2}$

18 s - 20 s $\quad \overline{a} = \dfrac{(0 - 24)\text{ m s}^{-1}}{(20 - 18)\text{ s}} = -12 \text{ m s}^{-2}$

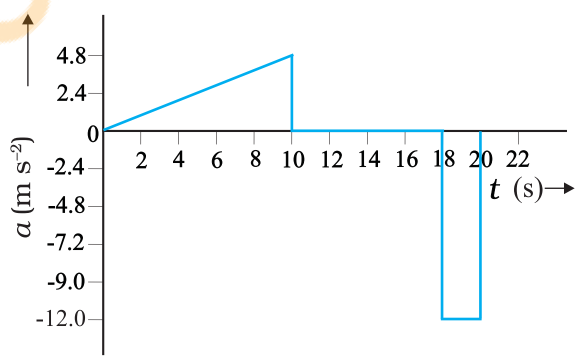

Fig. 3.8 *Acceleration as a function of time for motion represented in Fig. 3.3.*

Instantaneous acceleration is defined in the same way as the instantaneous velocity :

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \quad (3.5)$$

The acceleration at an instant is the slope of the tangent to the v–t curve at that instant. For the v–t curve shown in Fig. 3.7, we can obtain acceleration at every instant of time. The resulting $a - t$ curve is shown in Fig. 3.8. We see