|Displacement| = distance } only if the body moves in the
and |av. velocity| = av. speed } same direction throughout the motion

is equal to the average speed. This is not always the case, as you will see in the following example.

Example 3.1 A car is moving along a straight line, say OP in Fig. 3.1. It moves from O to P in 18 s and returns from P to Q in 6.0 s. What are the average velocity and average speed of the car in going (a) from O to P ? and (b) from O to P and back to Q ?

Answer (a)

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time interval}}$$

$$\bar{v} = \frac{+\,360\ \text{m}}{18\ \text{s}} = +\,20\ \text{m s}^{-1}$$

$$\text{Average speed} = \frac{\text{Path length}}{\text{Time interval}}$$

$$= \frac{360\ \text{m}}{18\ \text{s}} = 20\ \text{m s}^{-1}$$

Thus, in this case the average speed is equal to the magnitude of the average velocity.

(b) In this case,

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time interval}} = \frac{+240\ \text{m}}{(18+6.0)\ \text{s}}$$

$$= +10\ \text{m s}^{-1}$$

$$\text{Average speed} = \frac{\text{Path length}}{\text{Time interval}} = \frac{OP + PQ}{\Delta t}$$

$$= \frac{(360+120)\ \text{m}}{24\ \text{s}} = 20\ \text{m s}^{-1}$$

Thus, in this case the average speed is *not* equal to the magnitude of the average velocity. This happens because the motion here involves change in direction so that the path length is greater than the magnitude of displacement. This shows that **speed is, in general, greater than the magnitude of the velocity.**

If the car in Example 3.1 moves from O to P and comes back to O in the same time interval, average speed is 20 m/s but the average velocity is zero !

In general
|av. velocity| ≤ av. speed

3.4 INSTANTANEOUS VELOCITY AND SPEED

The average velocity tells us how fast an object has been moving over a given time interval but does not tell us how fast it moves at different instants of time during that interval. For this, we define **instantaneous velocity** or simply velocity v at an instant t.

The velocity at an instant is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small. In other words,

Mathematical Def. of Inst. velocity

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \tag{3.3a}$$

$$\boxed{v = \frac{dx}{dt}} \tag{3.3b}$$

where the symbol $\lim_{\Delta t \to 0}$ stands for the operation of taking limit as $\Delta t \to 0$ of the quantity on its right. In the language of calculus, the quantity on the right hand side of Eq. (3.3a) is the differential coefficient of x with respect to t and is denoted by $\frac{dx}{dt}$ (see Appendix 3.1). It is the rate of change of position with respect to time, at that instant.

We can use Eq. (3.3a) for obtaining the value of velocity at an instant either **graphically** or **numerically**. Suppose that we want to obtain graphically the value of velocity at time $t = 4$ s (point P) for the motion of the car represented in Fig. 3.3. The figure has been redrawn in Fig. 3.6 choosing different scales to facilitate the

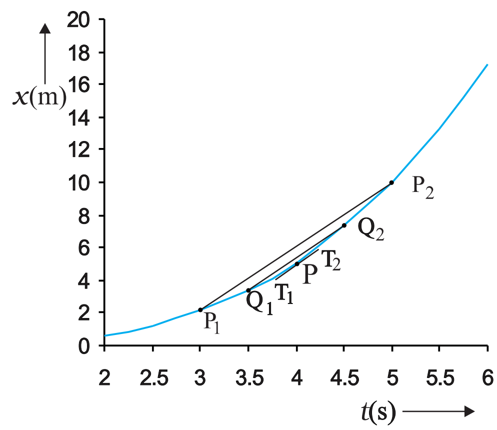

Fig. 3.6 *Determining velocity from position-time graph. Velocity at t = 4 s is the slope of the tangent to the graph at that instant.*