Explanation of Heisenberg's uncertainity principle from de-Broglie's hypothesis

wavelength extends all over space. By Born's probability interpretation this means that the electron is not localised in any finite region of space. That is, its position uncertainty is infinite ($\Delta x \to \infty$), which is consistent with the uncertainty principle.

In general, the matter wave associated with the electron is not extended all over space. It is a wave packet extending over some finite region of space. In that case Δx is not infinite but has some finite value depending on the extension of the wave packet. Also, you must appreciate that a wave packet of finite extension does not have a single wavelength. It is built up of wavelengths spread around some central wavelength.

By de Broglie's relation, then, the momentum of the electron will also have a spread – an uncertainty Δp. This is as expected from the uncertainty principle. It can be shown that the wave packet description together with de Broglie relation and Born's probability interpretation reproduce the Heisenberg's uncertainty principle exactly.

In Chapter 12, the de Broglie relation will be seen to justify Bohr's postulate on quantisation of angular momentum of electron in an atom.

Figure 11.6 shows a schematic diagram of (a) a localised wave packet, and (b) an extended wave with fixed wavelength.

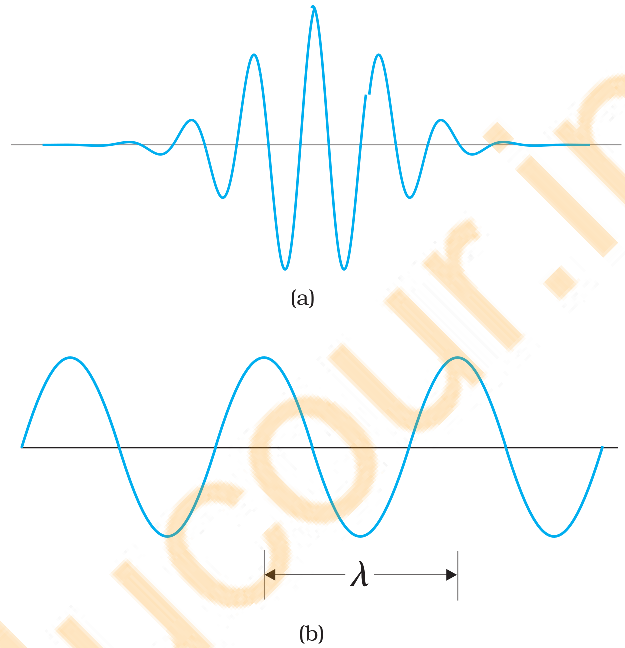

FIGURE 11.6 (a) The wave packet description of an electron. The wave packet corresponds to a spread of wavelength around some central wavelength (and hence by de Broglie relation, a spread in momentum). Consequently, it is associated with an uncertainty in position (Δx) and an uncertainty in momentum (Δp). (b) The matter wave corresponding to a definite momentum of an electron extends all over space. In this case, $\Delta p = 0$ and $\Delta x \to \infty$.

Example 11.4 What is the de Broglie wavelength associated with (a) an electron moving with a speed of 5.4×10^{6} m/s, and (b) a ball of mass 150 g travelling at 30.0 m/s?

Solution

(a) For the electron:

Mass $m = 9.11\times10^{-31}$ kg, speed $v = 5.4\times10^{6}$ m/s. Then, momentum

$p = m\,v = 9.11\times10^{-31}$ (kg) $\times$ 5.4 $\times$ 10^{6} (m/s)

$p = 4.92 \times 10^{-24}$ kg m/s

de Broglie wavelength, $\lambda = h/p$

$$= \frac{6.63\times10^{-34}\ \text{J s}}{4.92\times10^{-24}\ \text{kg m/s}}$$

$\lambda = 0.135$ nm

(b) For the ball:

Mass $m' = 0.150$ kg, speed $v' = 30.0$ m/s.

Then momentum $p' = m'\,v' = 0.150$ (kg) $\times$ 30.0 (m/s)

$p' = 4.50$ kg m/s

de Broglie wavelength $\lambda' = h/p'$.

EXAMPLE 11.4