Normal modes of a circular membrane rigidly clamped to the circumference as in a tabla are determined by the boundary condition that no point on the circumference of the membrane vibrates. Estimation of the frequencies of normal modes of this system is more complex. This problem involves wave propagation in two dimensions. However, the underlying physics is the same.

Example on open end organ pipe.

Example 15.5 A pipe, 30.0 cm long, is open at both ends. Which harmonic mode of the pipe resonates a 1.1 kHz source? Will resonance with the same source be observed if one end of the pipe is closed ? Take the speed of sound in air as 330 m s^{-1}.

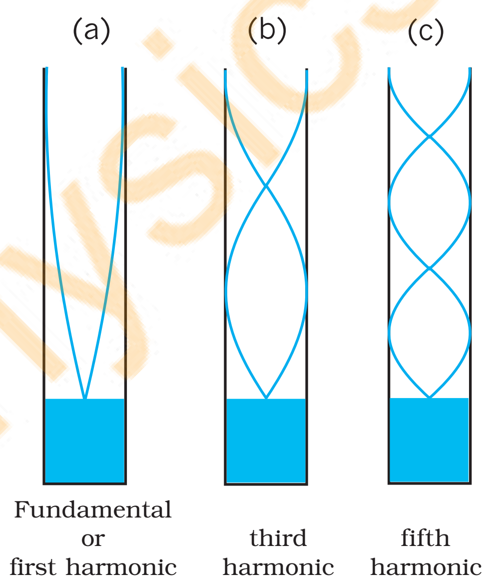
Answer The first harmonic frequency is given by

$$\boxed{\nu_1 = \frac{v}{\lambda_1} = \frac{v}{2L}} \quad \text{(open pipe)}$$

where L is the length of the pipe. The frequency of its nth harmonic is:

$$\nu_n = \frac{nv}{2L}, \text{ for } n = 1, 2, 3, \ldots \text{ (open pipe)}$$

First few modes of an open pipe are shown in Fig. 15.15.



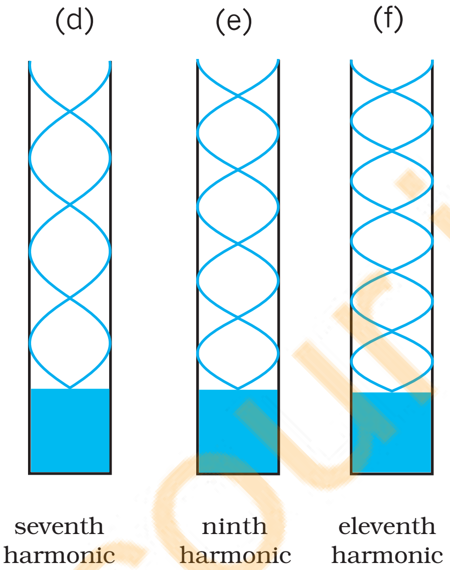


Fig. 15.14 *Normal modes of an air column open at one end and closed at the other end. Only the odd harmonics are seen to be possible.*

For L = 30.0 cm, v = 330 m s^{-1},

$$\nu_n = \frac{n\ 330\ (\text{m s}^{-1})}{0.6\ (\text{m})} = 550\ n\ \text{s}^{-1}$$

Clearly, a source of frequency 1.1 kHz will resonate at ν_2, i.e. the **second harmonic**.

For closed end organ pipe

Now if one end of the pipe is closed (Fig. 15.15), it follows from Eq. (14.50) that the fundamental frequency is

$$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{4L} \quad \text{(pipe closed at one end)}$$

and only the odd numbered harmonics are present :

$$\nu_3 = \frac{3v}{4L},\ \nu_5 = \frac{5v}{4L}, \text{ and so on.}$$

For L = 30 cm and v = 330 m s^{-1}, the fundamental frequency of the pipe closed at one end is 275 Hz and the source frequency corresponds to its fourth harmonic. Since this harmonic is not a possible mode, no resonance will be observed with the source, the moment one end is closed. ◀

15.7 BEATS

'Beats' is an interesting phenomenon arising from interference of waves. When two harmonic sound waves of close (but not equal) frequencies