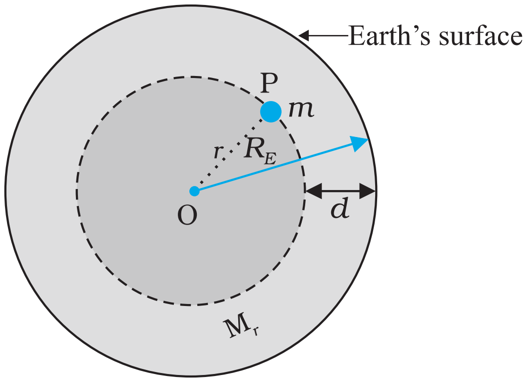


***Fig. 8.7** The mass m is in a mine located at a depth d below the surface of the Earth of mass M_E and radius R_E. We treat the Earth to be spherically symmetric.*

Again consider the earth to be made up of concentric shells as before and a point mass m situated at a distance r from the centre. The point P lies outside the sphere of radius r. For the shells of radius greater than r, the point P lies inside. Hence according to result stated in the last section, they exert no gravitational force on mass m kept at P. The shells with radius $\leq r$ make up a sphere of radius r for which the point P lies on the surface. This smaller sphere therefore exerts a force on a mass m at P as if its mass M_r is concentrated at the centre. Thus the force on the mass m at P has a magnitude

$$F = \frac{Gm\,(M_r)}{r^2} \tag{8.9}$$

We assume that the entire earth is of uniform density and hence its mass is $M_E = \frac{4\pi}{3} R_E^3\,\rho$ where M_E is the mass of the earth R_E is its radius and ρ is the density. On the other hand the mass of the sphere M_r of radius r is $\frac{4\pi}{3}\rho r^3$ and hence

$$F = G\,m\left(\frac{4p}{3}r\right)\frac{r^3}{r^2} = G\,m\left(\frac{M_E}{R_E^3}\right)\frac{r^3}{r^2}$$

$$\boxed{F = \frac{G\,m\,M_E}{R_E{}^3}\,r} \tag{8.10}$$

If the mass m is situated on the surface of earth, then $r = R_E$ and the gravitational force on it is, from Eq. (8.10)

$$\boxed{F = G\,\frac{M_E m}{R_E^2}} \tag{8.11}$$

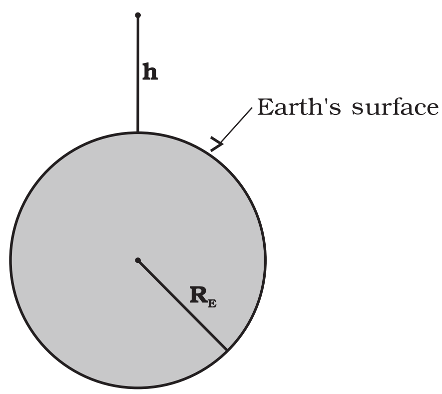
The acceleration experienced by the mass m, which is usually denoted by the symbol g is related to F by Newton's 2nd law by relation $F = mg$. Thus

$$\boxed{g = \frac{F}{m} = \frac{GM_E}{R_E^2}} \tag{8.12}$$

Acceleration g is readily measurable. R_E is a known quantity. The measurement of G by Cavendish's experiment (or otherwise), combined with knowledge of g and R_E enables one to estimate M_E from Eq. (8.12). This is the reason why there is a popular statement regarding Cavendish : "Cavendish weighed the earth".

8.6 ACCELERATION DUE TO GRAVITY BELOW AND ABOVE THE SURFACE OF EARTH

Consider a point mass m at a height h above the surface of the earth as shown in Fig. 8.8(a). The radius of the earth is denoted by R_E. Since this point is outside the earth,



(a)

***Fig. 8.8 (a)** g at a height h above the surface of the earth.*

its distance from the centre of the earth is $(R_E + h)$. If $F(h)$ denoted the magnitude of the force on the point mass m, we get from Eq. (8.5) :

$$F(h) = \frac{GM_E m}{(R_E + h)^2} \tag{8.13}$$

The acceleration experienced by the point mass is $F(h)/m \equiv g(h)$ and we get