$$= -\frac{2\,G\,m^2}{l}\left(2+\frac{1}{\sqrt{2}}\right) = -5.41\frac{G\,m^2}{l}$$

The gravitational potential at the centre of the square $\left(r=\sqrt{2}\,l/2\right)$ is

$$U(r) = -4\sqrt{2}\,\frac{G\,m}{l}.$$

8.8 ESCAPE SPEED

If a stone is thrown by hand, we see it falls back to the earth. Of course using machines we can shoot an object with much greater speeds and with greater and greater initial speed, the object scales higher and higher heights. A natural query that arises in our mind is the following: 'can we throw an object with such high initial speeds that it does not fall back to the earth?'

The principle of conservation of energy helps us to answer this question. Suppose the object did reach infinity and that its speed there was V_f. The energy of an object is the sum of potential and kinetic energy. As before W_1 denotes that gravitational potential energy of the object at infinity. The total energy of the projectile at infinity then is

$$E\,(\infty) = W_1 + \frac{mV_f^2}{2} \qquad (8.26)$$

If the object was thrown initially with a speed V_i from a point at a distance $(h+R_E)$ from the centre of the earth (R_E = radius of the earth), its energy initially was

$$E\,(h+R_E) = \frac{1}{2}mV_i^2 - \frac{GmM_E}{(h+R_E)} + W_1 \qquad (8.27)$$

By the principle of energy conservation Eqs. (8.26) and (8.27) must be equal. Hence

$$\frac{mV_i^2}{2} - \frac{GmM_E}{(h+R_E)} = \frac{mV_f^2}{2} \qquad (8.28)$$

The R.H.S. is a positive quantity with a minimum value zero hence so must be the L.H.S. Thus, an object can reach infinity as long as V_i is such that

$$\frac{mV_i^2}{2} - \frac{GmM_E}{(h+R_E)} \geq 0 \qquad (8.29)$$

The minimum value of V_i corresponds to the case when the L.H.S. of Eq. (8.29) equals zero.

Thus, the minimum speed required for an object to reach infinity (i.e. escape from the earth) corresponds to

$$\frac{1}{2}m\left(V_i^2\right)_{\min} = \frac{GmM_E}{h+R_E} \qquad (8.30)$$

If the object is thrown from the surface of the earth, h = 0, and we get

$$\left(V_i\right)_{\min} = \sqrt{\frac{2GM_E}{R_E}} \qquad (8.31)$$

Using the relation $g = GM_E/R_E^2$, we get

$$\left(V_i\right)_{\min} = \sqrt{2gR_E} \qquad (8.32)$$

Using the value of g and R_E, numerically $(V_i)_{\min} \approx 11.2$ km/s. This is called the escape speed, sometimes loosely called the escape velocity.

Equation (8.32) applies equally well to an object thrown from the surface of the moon with g replaced by the acceleration due to Moon's gravity on its surface and r_E replaced by the radius of the moon. Both are smaller than their values on earth and the escape speed for the moon turns out to be 2.3 km/s, about five times smaller. This is the reason that moon has no atmosphere. Gas molecules if formed on the surface of the moon having velocities larger than this will escape the gravitational pull of the moon.

Example 8.4 Two uniform solid spheres of equal radii R, but mass M and 4 M have a centre to centre separation 6 R, as shown in Fig. 8.10. The two spheres are held fixed. A projectile of mass m is projected from the surface of the sphere of mass M directly towards the centre of the second sphere. Obtain an expression for the minimum speed v of the projectile so that it reaches the surface of the second sphere.

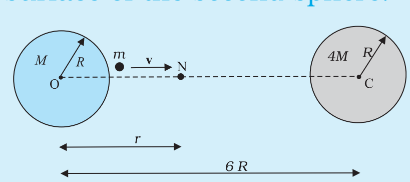

Fig. 8.10

Answer The projectile is acted upon by two mutually opposing gravitational forces of the two