problems without explicitly outlining and justifying the procedure. We now realise that in earlier studies we assumed, without saying so, that rotational motion and/or internal motion of the particles were either absent or negligible. We no longer need to do this. We have not only found the justification of the procedure we followed earlier; but we also have found how to describe and separate the translational motion of (1) a rigid body which may be rotating as well, or (2) a system of particles with all kinds of internal motion.

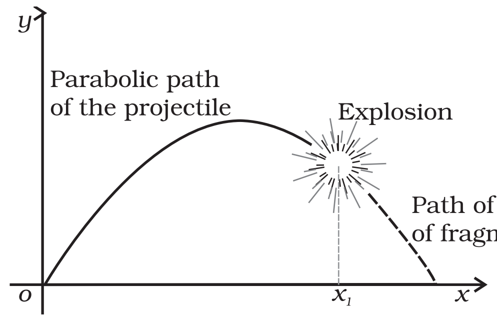

Fig. 7.12 *The centre of mass of the fragments of the projectile continues along the same parabolic path which it would have followed if there were no explosion.*

Direct questions are framed on this concept

Figure 7.12 is a good illustration of Eq. (7.11). A projectile, following the usual parabolic trajectory, explodes into fragments midway in air. The forces leading to the explosion are internal forces. They contribute nothing to the motion of the centre of mass. The total external force, namely, the force of gravity acting on the body, is the same before and after the explosion. The centre of mass under the influence of the external force continues, therefore, along the same parabolic trajectory as it would have followed if there were no explosion.

7.4 LINEAR MOMENTUM OF A SYSTEM OF PARTICLES

Let us recall that the linear momentum of a particle is defined as

$$\mathbf{p} = m\,\mathbf{v} \tag{7.12}$$

Let us also recall that Newton's second law written in symbolic form for a single particle is

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \tag{7.13}$$

where **F** is the force on the particle. Let us consider a system of n particles with masses m_1, m_2,...m_n respectively and velocities $\mathbf{v}_1, \mathbf{v}_2, \ldots\ldots, \mathbf{v}_n$ respectively. The particles may be interacting and have external forces acting on them. The linear momentum of the first particle is $m_1\mathbf{v}_1$, of the second particle is $m_2\mathbf{v}_2$ and so on.

For the system of n particles, the linear momentum of the system is defined to be the vector sum of all individual particles of the system,

$$\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2 + \ldots + \mathbf{p}_n$$

$$= m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + \ldots + m_n\mathbf{v}_n \tag{7.14}$$

Comparing this with Eq. (7.8)

$$\boxed{\mathbf{P} = M\,\mathbf{V}} \tag{7.15}$$

Thus, **the total momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its centre of mass**. Differentiating Eq. (7.15) with respect to time,

$$\frac{d\mathbf{P}}{dt} = M\frac{d\mathbf{V}}{dt} = M\mathbf{A} \tag{7.16}$$

Comparing Eq.(7.16) and Eq. (7.11),

$$\boxed{\frac{d\mathbf{P}}{dt} = \mathbf{F}_{ext}} \tag{7.17}$$

This is the statement of **Newton's second law of motion extended to a system of particles.**

Suppose now, that the sum of external forces acting on a system of particles is zero. Then from Eq.(7.17)

$$\frac{d\mathbf{P}}{dt} = 0 \quad \text{or} \quad \mathbf{P} = \text{Constant} \tag{7.18a}$$

Thus, when the total external force acting on a system of particles is zero, the total linear momentum of the system is constant. This is the law of conservation of the total linear momentum of a system of particles. Because of Eq. (7.15), this also means that when the total external force on the system is zero the velocity of the centre of mass remains constant. (We assume throughout the discussion on systems of particles in this chapter that the total mass of the system remains constant.)

Statement of Law of conservation of momentum

Statement of principle of conservation of velocity of C.M.

Note that on account of the internal forces, i.e. the forces exerted by the particles on one another, the individual particles may have