Take $m_1 \neq m_2 \neq m_3$, but use the same formula for X and Y.

of Fig. 7.11 had different masses. How will you then determine the centre of mass of the lamina?
◀

7.3 MOTION OF CENTRE OF MASS

Equipped with the definition of the centre of mass, we are now in a position to discuss its physical importance for a system of n particles. We may rewrite Eq.(7.4d) as

$$M\mathbf{R} = \sum m_i \mathbf{r}_i = m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + \ldots + m_n\mathbf{r}_n \quad (7.7)$$

Differentiating the two sides of the equation with respect to time we get

$$M\frac{d\mathbf{R}}{dt} = m_1\frac{d\mathbf{r}_1}{dt} + m_2\frac{d\mathbf{r}_2}{dt} + \ldots + m_n\frac{d\mathbf{r}_n}{dt}$$

or

$$M\mathbf{V} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + \ldots + m_n\mathbf{v}_n \quad (7.8)$$

where $\mathbf{v}_1 (= d\mathbf{r}_1/dt)$ is the velocity of the first particle $\mathbf{v}_2 (= d\mathbf{r}_2/dt)$ is the velocity of the second particle etc. and $\mathbf{V} = d\mathbf{R}/dt$ is the velocity of the centre of mass. Note that we assumed the masses m_1, m_2, ... etc. do not change in time. We have therefore, treated them as constants in differentiating the equations with respect to time.

Differentiating Eq.(7.8) with respect to time, we obtain

$$M\frac{d\mathbf{V}}{dt} = m_1\frac{d\mathbf{v}_1}{dt} + m_2\frac{d\mathbf{v}_2}{dt} + \ldots + m_n\frac{d\mathbf{v}_n}{dt}$$

or

$$M\mathbf{A} = m_1\mathbf{a}_1 + m_2\mathbf{a}_2 + \ldots + m_n\mathbf{a}_n \quad (7.9)$$

where $\mathbf{a}_1 (= d\mathbf{v}_1/dt)$ is the acceleration of the first particle, $\mathbf{a}_2 (= d\mathbf{v}_2/dt)$ is the acceleration of the second particle etc. and $\mathbf{A} (= d\mathbf{V}/dt)$ is the acceleration of the centre of mass of the system of particles.

Now, from Newton's second law, the force acting on the first particle is given by $\mathbf{F}_1 = m_1\mathbf{a}_1$. The force acting on the second particle is given by $\mathbf{F}_2 = m_2\mathbf{a}_2$ and so on. Eq. (7.9) may be written as

$$M\mathbf{A} = \mathbf{F}_1 + \mathbf{F}_2 + \ldots + \mathbf{F}_n \quad (7.10)$$

Thus, the total mass of a system of particles times the acceleration of its centre of mass is the vector sum of all the forces acting on the system of particles.

Note when we talk of the force $\mathbf{F}_1$ on the first particle, it is not a single force, but the vector sum of all the forces on the first particle; likewise for the second particle etc. Among these forces on each particle there will be **external** forces exerted by bodies outside the system and also **internal** forces exerted by the particles on one another. We know from Newton's third law that these internal forces occur in equal and opposite pairs and in the sum of forces of Eq. (7.10), their contribution is zero. Only the external forces contribute to the equation. We can then rewrite Eq. (7.10) as

$$M\mathbf{A} = \mathbf{F}_{ext} \quad (7.11)$$ ✓ Imp

where $\mathbf{F}_{ext}$ represents the sum of all external forces acting on the particles of the system.

Eq. (7.11) states that **the centre of mass of a system of particles moves as if all the mass of the system was concentrated at the centre of mass and all the external forces were applied at that point.**

Notice, to determine the motion of the centre of mass no knowledge of internal forces of the system of particles is required; for this purpose we need to know only the external forces.

To obtain Eq. (7.11) we did not need to specify the nature of the system of particles. The system may be a collection of particles in which there may be all kinds of internal motions, or it may be a rigid body which has either pure translational motion or a combination of translational and rotational motion. Whatever is the system and the motion of its individual particles, the centre of mass moves according to Eq. (7.11).

reason

Instead of treating extended bodies as single particles as we have done in earlier chapters, we can now treat them as systems of particles. We can obtain the translational component of their motion, i.e. the motion of the centre of mass of the system, by taking the mass of the whole system to be concentrated at the centre of mass and all the external forces on the system to be acting at the centre of mass. V. Imp.

This is the procedure that we followed earlier in analysing forces on bodies and solving