With the x–and y–axes chosen as shown in Fig. 7.9, the coordinates of points O, A and B forming the equilateral triangle are respectively (0,0), (0.5,0), $(0.25, 0.25\sqrt{3})$. Let the masses 100 g, 150g and 200g be located at O, A and B be respectively. Then,

$$X = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{100(0) + 150(0.5) + 200(0.25) \quad \text{g m}}{(100 + 150 + 200) \quad \text{g}}$$

$$= \frac{75 + 50}{450}\,\text{m} = \frac{125}{450}\,\text{m} = \frac{5}{18}\,\text{m}$$

$$Y = \frac{100(0) + 150(0) + 200(0.25\sqrt{3}) \quad \text{g m}}{450 \text{ g}}$$

$$= \frac{50\sqrt{3}}{450}\,\text{m} = \frac{\sqrt{3}}{9}\,\text{m} = \frac{1}{3\sqrt{3}}\,\text{m}$$

Because the masses are not symmetrically distributed in this case.

The centre of mass C is shown in the figure. Note that it is not the geometric centre of the triangle OAB. Why?

Example 7.2 Find the centre of mass of a triangular lamina.

Answer The lamina (ΔLMN) may be subdivided into narrow strips each parallel to the base (MN) as shown in Fig. 7.10

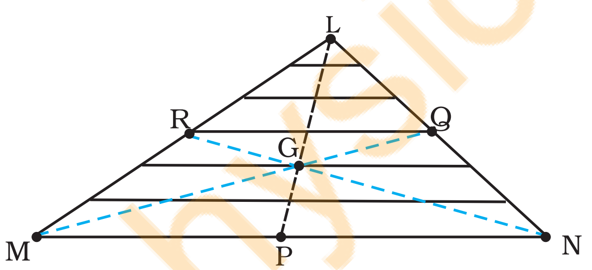

Fig. 7.10

By symmetry each strip has its centre of mass at its midpoint. If we join the midpoint of all the strips we get the median LP. The centre of mass of the triangle as a whole therefore, has to lie on the median LP. Similarly, we can argue that it lies on the median MQ and NR. This means the centre of mass lies on the point of concurrence of the medians, i.e. on the centroid G of the triangle.

Example 7.3 Find the centre of mass of a uniform L-shaped lamina (a thin flat plate) with dimensions as shown. The mass of the lamina is 3 kg.

Answer Choosing the X and Y axes as shown in Fig. 7.11 we have the coordinates of the vertices of the L-shaped lamina as given in the figure. We can think of the L-shape to consist of 3 squares each of length 1m. The mass of each square is 1kg, since the lamina is uniform. The centres of mass C_1, C_2 and C_3 of the squares are, by symmetry, their geometric centres and have coordinates (1/2,1/2), (3/2,1/2), (1/2,3/2) respectively. We take the masses of the squares to be concentrated at these points. The centre of mass of the whole L shape (X, Y) is the centre of mass of these mass points.

Here the small squares are being replaced by point objects, kept at their respective centre of masses.

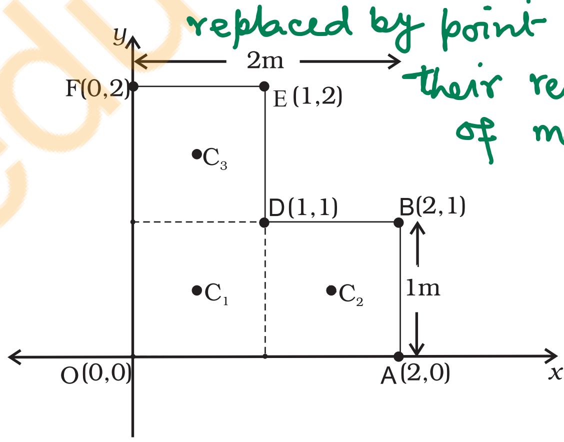

Fig. 7.11

Hence

$$X = \frac{[1(1/2) + 1(3/2) + 1(1/2)]\,\text{kg m}}{(1+1+1)\,\text{kg}} = \frac{5}{6}\,\text{m}$$

$$Y = \frac{[1(1/2) + 1(1/2) + 1(3/2)] \quad \text{kg m}}{(1+1+1)\,\text{kg}} = \frac{5}{6}\,\text{m}$$

The centre of mass of the L-shape lies on the line OD. We could have guessed this without calculations. Can you tell why? Suppose, the three squares that make up the L shaped lamina