

15. Rotation about a fixed axis is directly analogous to linear motion in respect of kinematics and dynamics.
16. For a rigid body rotating about a fixed axis (say,  $z$ -axis) of rotation,  $L_z = I\omega$ , where  $I$  is the moment of inertia about  $z$ -axis. In general, the angular momentum  $\mathbf{L}$  for such a body is not along the axis of rotation. Only if the body is symmetric about the axis of rotation,  $\mathbf{L}$  is along the axis of rotation. In that case,  $|\mathbf{L}| = L_z = I\omega$ . The angular acceleration of a rigid body rotating about a fixed axis is given by  $I\alpha = \tau$ . If the external torque  $\tau$  acting on the body is zero, the component of angular momentum about the fixed axis (say,  $z$ -axis),  $L_z (=I\omega)$  of such a rotating body is constant.
17. For rolling motion without slipping  $v_{cm} = R\omega$ , where  $v_{cm}$  is the velocity of translation (i.e. of the centre of mass),  $R$  is the radius and  $m$  is the mass of the body. The kinetic energy of such a rolling body is the sum of kinetic energies of translation and rotation:

$$K = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2 .$$

Quantity	Symbols	Dimensions	Units	Remarks
Angular velocity	$\omega$	$[T^{-1}]$	rad $s^{-1}$	$\mathbf{v} = \omega \times \mathbf{r}$
Angular momentum	$\mathbf{L}$	$[ML^2 T^{-1}]$	J s	$\mathbf{L} = \mathbf{r} \times \mathbf{p}$
Torque	$\boldsymbol{\tau}$	$[ML^2 T^{-2}]$	N m	$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$
Moment of inertia	$I$	$[ML^2]$	kg $m^2$	$I = \sum m_i r_i^2$

#### POINTS TO PONDER

- To determine the motion of the centre of mass of a system no knowledge of internal forces of the system is required. For this purpose we need to know only the external forces on the body.
- Separating the motion of a system of particles as the motion of the centre of mass, (i.e., the translational motion of the system) and motion about (i.e. relative to) the centre of mass of the system is a useful technique in dynamics of a system of particles. One example of this technique is separating the kinetic energy of a system of particles  $K$  as the kinetic energy of the system about its centre of mass  $K'$  and the kinetic energy of the centre of mass  $MV^2/2$ ,  

$$K = K' + MV^2/2$$
- Newton's Second Law for finite sized bodies (or systems of particles) is based in Newton's Second Law and also Newton's Third Law for particles.
- To establish that the time rate of change of the total angular momentum of a system of particles is the total external torque in the system, we need not only Newton's second law for particles, but also Newton's third law with the provision that the forces between any two particles act along the line joining the particles.
- The vanishing of the total external force and the vanishing of the total external torque are independent conditions. We can have one without the other. In a couple, total external force is zero, but total torque is non-zero.
- The total torque on a system is independent of the origin if the total external force is zero.
- The centre of gravity of a body coincides with its centre of mass only if the gravitational field does not vary from one part of the body to the other.
- The angular momentum  $\mathbf{L}$  and the angular velocity  $\boldsymbol{\omega}$  are not necessarily parallel vectors. However, for the simpler situations discussed in this chapter when rotation is about a fixed axis which is an axis of symmetry of the rigid body, the relation  $\mathbf{L} = I\boldsymbol{\omega}$  holds good, where  $I$  is the moment of the inertia of the body about the rotation axis.