Equation (7.49b) applies to any rolling body: a disc, a cylinder, a ring or a sphere.

Example 7.16 Three bodies, a ring, a solid cylinder and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of the bodies are identical. Which of the bodies reaches the ground with maximum velocity?

Answer We assume conservation of energy of the rolling body, i.e. there is no loss of energy due to friction etc. The potential energy lost by the body in rolling down the inclined plane (= mgh) must, therefore, be equal to kinetic energy gained. (See Fig.7.38) Since the bodies start from rest the kinetic energy gained is equal to the final kinetic energy of the bodies. From Eq. (7.49b), $K = \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$, where v is the final velocity of (the centre of mass of) the body. Equating K and mgh,

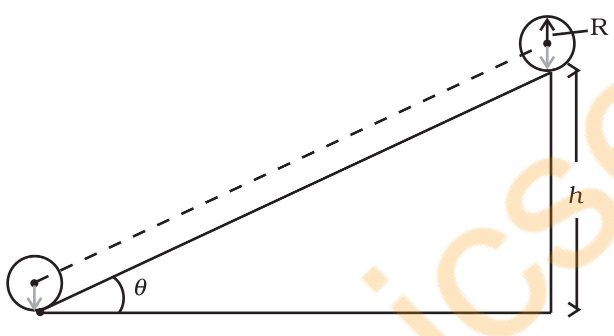

Fig.7.38

$$mgh = \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$$

$$\text{or } v^2 = \left(\frac{2gh}{1+k^2/R^2}\right)$$

Note is independent of the mass of the rolling body;

For a ring, $k^2 = R^2$

$$v_{ring} = \sqrt{\frac{2gh}{1+1}},$$

$$= \sqrt{gh}$$

For a solid cylinder $k^2 = R^2/2$

$$v_{disc} = \sqrt{\frac{2gh}{1+1/2}}$$

$$= \sqrt{\frac{4gh}{3}}$$

For a solid sphere $k^2 = 2R^2/5$

$$v_{sphere} = \sqrt{\frac{2gh}{1+2/5}}$$

$$= \sqrt{\frac{10gh}{7}}$$

From the results obtained it is clear that among the three bodies the sphere has the greatest and the ring has the least velocity of the centre of mass at the bottom of the inclined plane.

Suppose the bodies have the same mass. Which body has the greatest rotational kinetic energy while reaching the bottom of the inclined plane? ◀

SUMMARY

1. Ideally, a rigid body is one for which the distances between different particles of the body do not change, even though there are forces on them.
2. A rigid body fixed at one point or along a line can have only rotational motion. A rigid body not fixed in some way can have either pure translational motion or a combination of translational and rotational motions.
3. In rotation about a fixed axis, every particle of the rigid body moves in a circle which lies in a plane perpendicular to the axis and has its centre on the axis. Every Point in the rotating rigid body has the same angular velocity at any instant of time.
4. In pure translation, every particle of the body moves with the same velocity at any instant of time.
5. Angular velocity is a vector. Its magnitude is $\omega = d\theta/dt$ and it is directed along the axis of rotation. For rotation about a fixed axis, this vector $\boldsymbol{\omega}$ has a fixed direction.

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