which is in contact with the surface is at rest on the surface.

We have remarked earlier that rolling motion is a combination of rotation and translation. We know that the translational motion of a system of particles is the motion of its centre of mass.

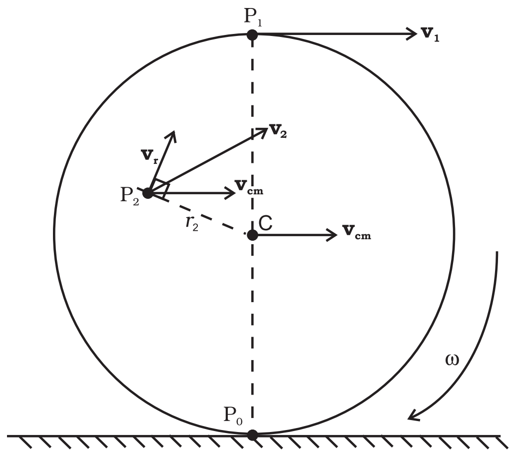

Fig. 7.37 *The rolling motion (without slipping) of a disc on a level surface. Note at any instant, the point of contact P_0 of the disc with the surface is at rest; the centre of mass of the disc moves with velocity, v_{cm}. The disc rotates with angular velocity ω about its axis which passes through C; $v_{cm} = R\omega$, where R is the radius of the disc.*

Let $\mathbf{v}_{cm}$ be the velocity of the centre of mass and therefore the translational velocity of the disc. Since the centre of mass of the rolling disc is at its geometric centre C (Fig. 7. 37), $\mathbf{v}_{cm}$ is the velocity of C. It is parallel to the level surface. The rotational motion of the disc is about its symmetry axis, which passes through C. Thus, the velocity of any point of the disc, like P$_0$, P$_1$ or P$_2$, consists of two parts, one is the translational velocity $\mathbf{v}_{cm}$ and the other is the linear velocity $\mathbf{v}_r$ on account of rotation. The magnitude of $\mathbf{v}_r$ is $v_r = r\omega$, where ω is the angular velocity of the rotation of the disc about the axis and r is the distance of the point from the axis (i.e. from C). The velocity $\mathbf{v}_r$ is directed perpendicular to the radius vector of the given point with respect to C. In Fig. 7.37, the velocity of the point P$_2$ ($\mathbf{v}_2$) and its components $\mathbf{v}_r$ and $\mathbf{v}_{cm}$ are shown; $\mathbf{v}_r$ here is perpendicular to CP$_2$. It is easy to show that $\mathbf{v}_2$ is perpendicular to the line P$_0$P$_2$. Therefore the line passing through P$_0$ and parallel to $\boldsymbol{\omega}$ is called the instantaneous axis of rotation.

At P$_0$, the linear velocity, $\mathbf{v}_r$, due to rotation is directed exactly opposite to the translational velocity $\mathbf{v}_{cm}$. Further the magnitude of $\mathbf{v}_r$ here is $R\omega$, where R is the radius of the disc. The condition that P$_0$ is instantaneously at rest requires $v_{cm} = R\omega$. Thus for the disc the condition for rolling without slipping is } V.imp

$$\boxed{v_{cm} = R\omega} \qquad (7.47)$$

Incidentally, this means that the velocity of point P$_1$ at the top of the disc ($\mathbf{v}_1$) has a magnitude $v_{cm} + R\omega$ or $2\,v_{cm}$ and is directed parallel to the level surface. The condition (7.47) applies to all rolling bodies.

7.14.1 Kinetic Energy of Rolling Motion

Our next task will be to obtain an expression for the kinetic energy of a rolling body. The kinetic energy of a rolling body can be separated into kinetic energy of translation and kinetic energy of rotation. This is a special case of a general result for a system of particles, according to which the kinetic energy of a system of particles (K) can be separated into the kinetic energy of translational motion of the centre of mass ($MV^2/2$) and kinetic energy of rotational motion about the centre of mass of the system of particles (K'). Thus,

$K = K_{Tr} + K_{rot}$

$$K = K' + MV^2/2 \qquad (7.48)$$

We assume this general result (see Exercise 7.31), and apply it to the case of rolling motion. In our notation, the kinetic energy of the centre of mass, i.e., the kinetic energy of translation, of the rolling body is $mv^2_{cm}/2$, where m is the mass of the body and v_{cm} is the centre of the mass velocity. Since the motion of the rolling body about the centre of mass is rotation, K' represents the kinetic energy of rotation of the body; $K' = I\omega^2/2$, where I is the moment of inertia about the appropriate axis, which is the symmetry axis of the rolling body. The kinetic energy of a rolling body, therefore, is given by

$K_{Tr} = \frac{1}{2}mv^2_{cm}$

$K_{rot} = \frac{1}{2}I\omega^2$

$$\boxed{K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2_{cm}} \rightarrow K = K_{Tr} + K_{rot} \qquad (7.49a)$$

Substituting $I = mk^2$ where k = the corresponding radius of gyration of the body and $v_{cm} = R\,\omega$, we get

$$K = \frac{1}{2}\frac{mk^2 v^2_{cm}}{R^2} + \frac{1}{2}mv^2_{cm}$$

$$\text{or} \quad \boxed{K = \frac{1}{2}mv^2_{cm}\left(1 + \frac{k^2}{R^2}\right)} \quad \text{V.imp} \qquad (7.49b)$$