$$\tau = I\alpha \qquad (7.43)$$

Eq. (7.43) is similar to Newton's second law for linear motion expressed symbolically as

$F = ma$

Just as force produces acceleration, torque produces angular acceleration in a body. The angular acceleration is directly proportional to the applied torque and is inversely proportional to the moment of inertia of the body. In this respect, Eq.(7.43) can be called Newton's second law for rotational motion about a fixed axis.

Example 7.15 A cord of negligible mass is wound round the rim of a fly wheel of mass 20 kg and radius 20 cm. A steady pull of 25 N is applied on the cord as shown in Fig. 7.35. The flywheel is mounted on a horizontal axle with frictionless bearings.

(a) Compute the angular acceleration of the wheel.

(b) Find the work done by the pull, when 2m of the cord is unwound.

(c) Find also the kinetic energy of the wheel at this point. Assume that the wheel starts from rest.

(d) Compare answers to parts (b) and (c).

Answer

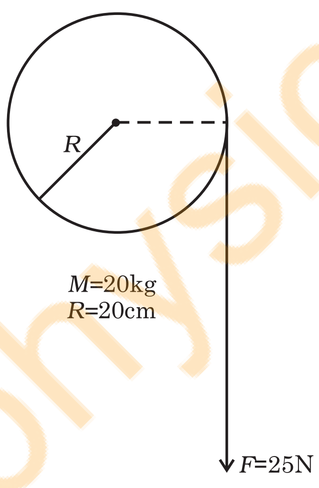

Fig. 7.35

(a) We use $I\alpha = \tau$

the torque $\tau = FR$

$= 25 \times 0.20$ Nm (as $R = 0.20$m)

$= 5.0$ Nm

I = Moment of inertia of flywheel about its axis $= \dfrac{MR^2}{2}$

$= \dfrac{20.0 \times (0.2)^2}{2} = 0.4$ kg m^2

α = angular acceleration

$= 5.0$ N m/0.4 kg m^2 = 12.5 s^{-2}

(b) Work done by the pull unwinding 2m of the cord

= 25 N × 2m = 50 J

(c) Let ω be the final angular velocity. The kinetic energy gained $= \dfrac{1}{2} I\omega^2$, since the wheel starts from rest. Now,

$\omega^2 = \omega_0^2 + 2\alpha\theta, \quad \omega_0 = 0$

The angular displacement θ = length of unwound string / radius of wheel

= 2m/0.2 m = 10 rad

$\omega^2 = 2 \times 12.5 \times 10.0 = 250\,(\text{rad/s})^2$

$\therefore$ K.E. gained $= \dfrac{1}{2} \times 0.4 \times 250 = 50$ J

(d) The answers are the same, i.e. the kinetic energy gained by the wheel = work done by the force. There is no loss of energy due to friction.

7.13 ANGULAR MOMENTUM IN CASE OF ROTATION ABOUT A FIXED AXIS

We have studied in section 7.7, the angular momentum of a system of particles. We already know from there that the time rate of total angular momentum of a system of particles about a point is equal to the total external torque on the system taken about the same point. When the total external torque is zero, the total angular momentum of the system is conserved.

We now wish to study the angular momentum in the special case of rotation about a fixed axis. The general expression for the total angular momentum of the system of n particles is

$$\mathbf{L} = \sum_{i=1}^{N} \mathbf{r}_i \times \mathbf{p}_i \qquad (7.25b)$$

We first consider the angular momentum of a typical particle of the rotating rigid body. We then sum up the contributions of individual particles to get $\mathbf{L}$ of the whole body.

For a typical particle $\mathbf{l} = \mathbf{r} \times \mathbf{p}$. As seen in the last section $\mathbf{r} = \mathbf{OP} = \mathbf{OC} + \mathbf{CP}$ [Fig. 7.17(b)]. With