Table 7.2 Comparison of Translational and Rotational Motion

	Linear Motion	Rotational Motion about a Fixed Axis
1	Displacement x	Angular displacement θ
2	Velocity $v = dx/dt$	Angular velocity $\omega = d\theta/dt$
3	Acceleration $a = dv/dt$	Angular acceleration $\alpha = d\omega/dt$
4	Mass M	Moment of inertia I
5	Force $F = Ma$	Torque $\tau = I\,\alpha$
6	Work $dW = F\,ds$	Work $W = \tau\, d\theta$
7	Kinetic energy $K = Mv^2/2$	Kinetic energy $K = I\omega^2/2$
8	Power $P = F\,v$	Power $P = \tau\omega$
9	Linear momentum $p = Mv$	Angular momentum $L = I\omega$

at P_1, and α_1 is the angle between $\mathbf{F}_1$ and the radius vector $\mathbf{OP}_1$; $\phi_1 + \alpha_1 = 90°$.

The torque due to $\mathbf{F}_1$ about the origin is $\mathbf{OP}_1 \times \mathbf{F}_1$. Now $\mathbf{OP}_1 = \mathbf{OC} + \mathbf{OP}_1$. [Refer to Fig. 7.17(b).] Since $\mathbf{OC}$ is along the axis, the torque resulting from it is excluded from our consideration. The effective torque due to $\mathbf{F}_1$ is $\boldsymbol{\tau}_1 = \mathbf{CP} \times \mathbf{F}_1$; it is directed along the axis of rotation and has a magnitude $\tau_1 = r_1 F_1 \sin\alpha$, Therefore,

$$dW_1 = \tau_1 d\theta$$

If there are more than one forces acting on the body, the work done by all of them can be added to give the total work done on the body. Denoting the magnitudes of the torques due to the different forces as $\tau_1, \tau_2, \ldots$ etc,

$$dW = (\tau_1 + \tau_2 + \ldots)d\theta$$

Remember, the forces giving rise to the torques act on different particles, but the angular displacement $d\theta$ is the same for all particles. Since all the torques considered are parallel to the fixed axis, the magnitude τ of the total torque is just the algebraic sum of the magnitudes of the torques, i.e., $\tau = \tau_1 + \tau_2 + \ldots$.. We, therefore, have

$$dW = \tau d\theta \qquad (7.41)$$

This expression gives the work done by the total (external) torque τ which acts on the body rotating about a fixed axis. Its similarity with the corresponding expression

$$dW = F\,ds$$

for linear (translational) motion is obvious.

Dividing both sides of Eq. (7.41) by dt gives

$$P = \frac{dW}{dt} = \tau\frac{d\theta}{dt} = \tau\omega$$

or $P = \tau\omega$ (7.42)

This is the instantaneous power. Compare this expression for power in the case of rotational motion about a fixed axis with that of power in the case of linear motion,

$$P = Fv$$

In a perfectly rigid body there is no internal motion. The work done by external torques is therefore, not dissipated and goes on to increase the kinetic energy of the body. The rate at which work is done on the body is given by Eq. (7.42). This is to be equated to the rate at which kinetic energy increases. The rate of increase of kinetic energy is

$$\frac{d}{dt}\left(\frac{I\omega^2}{2}\right) = I\frac{(2\omega)}{2}\frac{d\omega}{dt}$$

We assume that the moment of inertia does not change with time. This means that the mass of the body does not change, the body remains rigid and also the axis does not change its position with respect to the body.

Since $\alpha = d\omega/dt$, we get

$$\frac{d}{dt}\left(\frac{I\omega^2}{2}\right) = I\,\omega\,\alpha$$

Equating rates of work done and of increase in kinetic energy,

$$\tau\omega = I\,\omega\,\alpha$$