The angular acceleration of the engine $= 4\pi$ rad/s^2

(ii) The angular displacement in time t is given by

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$= (40\pi \times 16 + \frac{1}{2} \times 4\pi \times 16^2) \text{ rad}$$

$$= (640\pi + 512\pi) \text{ rad}$$

$$= 1152\pi \text{ rad}$$

Number of revolutions $= \dfrac{1152\pi}{2\pi} = 576$ ◀

7.12 DYNAMICS OF ROTATIONAL MOTION ABOUT A FIXED AXIS

Table 7.2 lists quantities associated with linear motion and their analogues in rotational motion. We have already compared kinematics of the two motions. Also, we know that in rotational motion moment of inertia and torque play the same role as mass and force respectively in linear motion. Given this we should be able to guess what the other analogues indicated in the table are. For example, we know that in linear motion, work done is given by $F\,dx$, in rotational motion about a fixed axis it should be $\tau d\theta$, since we already know the correspondence $dx \rightarrow d\theta$ and $F \rightarrow \tau$. It is, however, necessary that these correspondences are established on sound dynamical considerations. This is what we now turn to.

Most of the problems we get at this level are of rotational motion about fixed axis.

Before we begin, we note a **simplification that arises in the case of rotational motion about a fixed axis**. Since the axis is fixed, only those components of torques, which are along the direction of the fixed axis need to be considered in our discussion. Only these components can cause the body to rotate about the axis. 1 A component of the torque perpendicular to the axis of rotation will tend to turn the axis from its position. We specifically assume that there will arise necessary forces of constraint to cancel the effect of the perpendicular components of the (external) torques, so that the fixed position of the axis will be maintained. The perpendicular components of the torques, therefore need not be taken into account. This means that for our calculation of torques on a rigid body:

(1) We need to consider only those forces that lie in planes perpendicular to the axis. Forces which are parallel to the axis will give torques perpendicular to the axis and need not be taken into account.

(2) We need to consider only those components of the position vectors which are perpendicular to the axis. Components of position vectors along the axis will result in torques perpendicular to the axis and need not be taken into account.

Work done by a torque

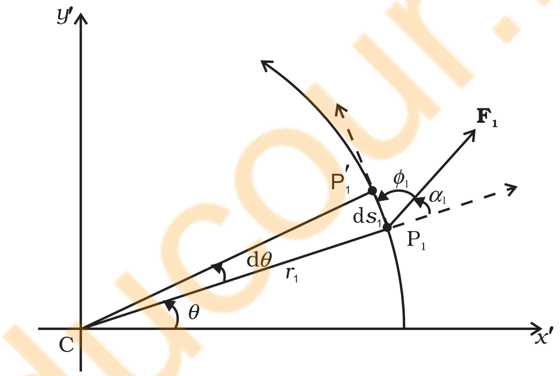

Fig. 7.34 *Work done by a force* $\mathbf{F}_1$ *acting on a particle of a body rotating about a fixed axis; the particle describes a circular path with centre C on the axis; arc* $P_1P'_1(ds_1)$ *gives the displacement of the particle.*

Figure 7.34 shows a cross-section of a rigid body rotating about a fixed axis, which is taken as the z-axis (perpendicular to the plane of the page; see Fig. 7.33). As said above we need to consider only those forces which lie in planes perpendicular to the axis. Let $\mathbf{F}_1$ be one such typical force acting as shown on a particle of the body at point P_1 with its line of action in a plane perpendicular to the axis. For convenience we call this to be the x'–y' plane (coincident with the plane of the page). The particle at P_1 describes a circular path of radius r_1 with centre C on the axis; $CP_1 = r_1$.

In time Δt, the point moves to the position P_1'. The displacement of the particle $d\mathbf{s}_1$, therefore, has magnitude $ds_1 = r_1 d\theta$ and direction tangential at P_1 to the circular path as shown. Here $d\theta$ is the angular displacement of the particle, $d\theta = \angle P_1CP_1'$. The work done by the force on the particle is

$$dW_1 = \mathbf{F}_1 . d\mathbf{s}_1 = F_1 ds_1 \cos\phi_1 = F_1 (r_1\, d\theta) \sin\alpha_1$$

where ϕ_1 is the angle between $\mathbf{F}_1$ and the tangent