there is no need to treat angular velocity as a vector. Further, the angular acceleration, $\alpha = \mathrm{d}\omega/\mathrm{d}t$.

Similarity between Translation motion and rotational motion

The kinematical quantities in rotational motion, angular displacement (θ), angular velocity (ω) and angular acceleration (α) respectively are analogous to kinematic quantities in linear motion, displacement (x), velocity (v) and acceleration (a). We know the kinematical equations of linear motion with uniform (i.e. constant) acceleration:

$$v = v_0 + at \tag{a}$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \tag{b}$$

$$v^2 = v_0^2 + 2ax \tag{c}$$

where x_0 = initial displacement and v_0= initial velocity. The word 'initial' refers to values of the quantities at $t = 0$

The corresponding kinematic equations for rotational motion with uniform angular acceleration are:

$$\omega = \omega_0 + \alpha t \tag{7.38}$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \tag{7.39}$$

$$\text{and } \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \tag{7.40}$$

where θ_0= initial angular displacement of the rotating body, and ω_0 = initial angular velocity of the body.

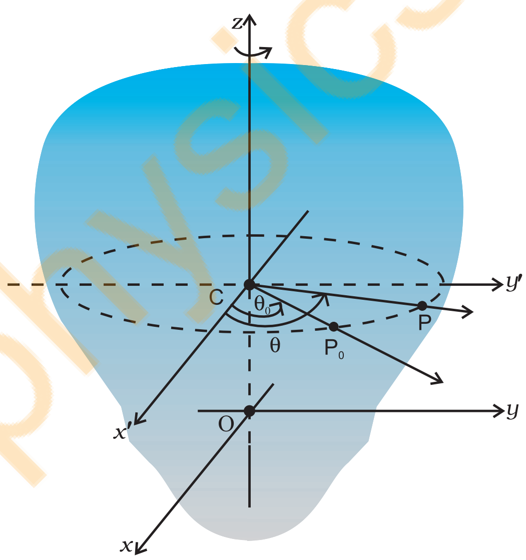

Fig.7.33 *Specifying the angular position of a rigid body.*

Example 7.13 Obtain Eq. (7.38) from first principles.

Answer The angular acceleration is uniform, hence

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \alpha = constant \tag{i}$$

Integrating this equation,

$$\omega = \int \alpha \,\mathrm{d}t + c$$

$$= \alpha t + c \quad (\text{as } \alpha \text{ is constant})$$

At $t = 0$, $\omega = \omega_0$ (given)

From (i) we get at $t = 0$, $\omega = c = \omega_0$

Thus, $\omega = \alpha t + \omega_0$ as required.

With the definition of $\omega = \mathrm{d}\theta/\mathrm{d}t$ we may integrate Eq. (7.38) to get Eq. (7.39). This derivation and the derivation of Eq. (7.40) is left as an exercise.

Example 7.14 The angular speed of a motor wheel is increased from 1200 rpm to 3120 rpm in 16 seconds. (i) What is its angular acceleration, assuming the acceleration to be uniform? (ii) How many revolutions does the engine make during this time?

Answer

(i) We shall use $\omega = \omega_0 + \alpha t$

ω_0 = initial angular speed in rad/s

$= 2\pi \times$ angular speed in rev/s

$$= \frac{2\pi \times \text{angular speed in rev/min}}{60 \text{ s/min}}$$

$$= \frac{2\pi \times 1200}{60} \text{ rad/s}$$

$= 40\pi$ rad/s

Similarly ω = final angular speed in rad/s

$$= \frac{2\pi \times 3120}{60} \text{ rad/s}$$

$= 2\pi \times 52$ rad/s

$= 104\,\pi$ rad/s

$\therefore$ Angular acceleration

$$\alpha = \frac{\omega - \omega_0}{t} = 4\,\pi \text{ rad/s}^2$$