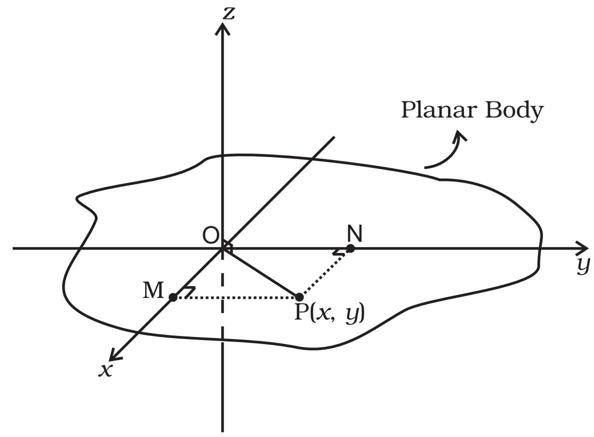

Fig. 7.29 *Theorem of perpendicular axes applicable to a planar body; x and y axes are two perpendicular axes in the plane and the z-axis is perpendicular to the plane.*

The figure shows a planar body. An axis perpendicular to the body through a point O is taken as the *z*-axis. Two mutually perpendicular axes lying in the plane of the body and concurrent with *z*-axis, i.e., passing through O, are taken as the *x* and *y*-axes. The theorem states that

$$\boxed{I_z = I_x + I_y} \qquad (7.36)$$

Let us look at the usefulness of the theorem through an example.

▶ **Example 7.10** What is the moment of inertia of a disc about one of its diameters?

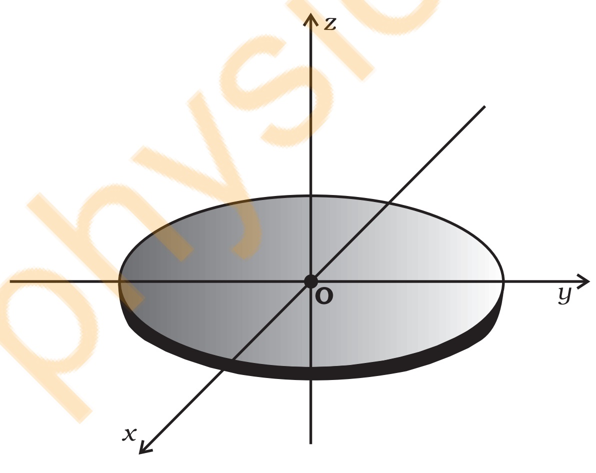

Fig. 7.30 *Moment of inertia of a disc about a diameter, given its moment of inertia about the perpendicular axis through its centre.*

Answer We assume the moment of inertia of the disc about an axis perpendicular to it and through its centre to be known; it is $MR^2/2$, where M is the mass of the disc and *R* is its radius (Table 7.1)

The disc can be considered to be a planar body. Hence the theorem of perpendicular axes is applicable to it. As shown in Fig. 7.30, we take three concurrent axes through the centre of the disc, O, as the *x*–, *y*– and *z*–axes; *x*– and *y*–axes lie in the plane of the disc and *z*–axis is perpendicular to it. By the theorem of perpendicular axes,

$$I_z = I_x + I_y$$

Now, *x* and *y* axes are along two diameters of the disc, and by symmetry the moment of inertia of the disc is the same about any diameter. Hence

$$I_x = I_y$$

and $I_z = 2I_x$

But $I_z = MR^2/2$

So finally, $I_x = I_z/2 = MR^2/4$

Thus the moment of inertia of a disc about any of its diameter is $MR^2/4$. ◀

Find similarly the moment of inertia of a ring about any of its diameters. Will the theorem be applicable to a solid cylinder?

axis passing through c.m of the body

axis about which M.I. has to be determined

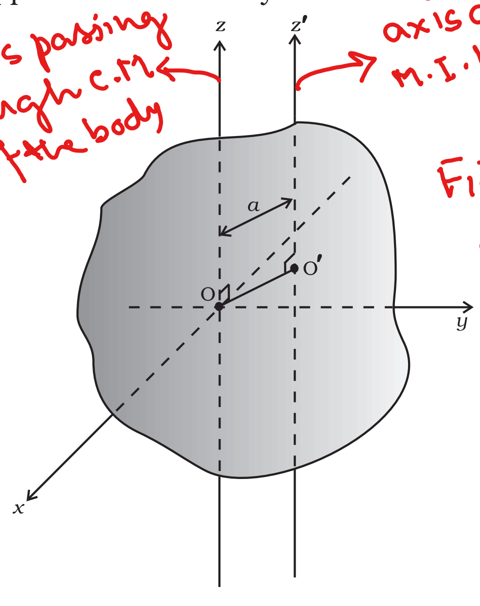

Fig for theorem of parallel axes

Fig.7.31 *The theorem of parallel axes The z and z′ axes are two parallel axes separated by a distance a; O is the centre of mass of the body, OO′ = a.*