Pythagoras theorem, BC = $2\sqrt{2}$ m. The forces on the ladder are its weight W acting at its centre of gravity D, reaction forces F_1 and F_2 of the wall and the floor respectively. Force F_1 is perpendicular to the wall, since the wall is frictionless. Force F_2 is resolved into two components, the normal reaction N and the force of friction F. Note that F prevents the ladder from sliding away from the wall and is therefore directed toward the wall.

For translational equilibrium, taking the forces in the vertical direction,

$$N - W = 0 \qquad \text{(i)}$$

Taking the forces in the horizontal direction,

$$F - F_1 = 0 \qquad \text{(ii)}$$

For rotational equilibrium, taking the moments of the forces about A,

$$2\sqrt{2}\,F_1 - (1/2)\ W = 0 \qquad \text{(iii)}$$

Now $W = 20\ g = 20 \times 9.8\ \text{N} = 196.0\ \text{N}$

From (i) $N = 196.0\ \text{N}$

From (iii) $F_1 = W/4\sqrt{2} = 196.0/4\sqrt{2} = 34.6\ \text{N}$

From (ii) $F = F_1 = 34.6\ \text{N}$

$$F_2 = \sqrt{F^2 + N^2} = 199.0\ \text{N}$$

The force F_2 makes an angle α with the horizontal,

$$\tan\alpha = N/F = 4\sqrt{2}\ , \quad \alpha = \tan^{-1}(4\sqrt{2}) \approx 80^\circ$$ ◀

7.9 MOMENT OF INERTIA

We have already mentioned that we are developing the study of rotational motion parallel to the study of translational motion with which we are familiar. We have yet to answer one major question in this connection. **What is the analogue of mass in rotational motion?** We shall attempt to answer this question in the present section. To keep the discussion simple, we shall consider rotation about a fixed axis only. Let us try to get an expression for *the* **kinetic energy of a rotating body**. We know that for a body rotating about a fixed axis, each particle of the body moves in a circle with linear velocity given by Eq. (7.19). (Refer to Fig. 7.16). For a particle at a distance from the axis, the linear velocity is $v_i = r_i\omega$. The kinetic energy of motion of this particle is

$$k_i = \frac{1}{2}m_i v_i^2 = \frac{1}{2}m_i r_i^2\omega^2$$

where m_i is the mass of the particle. The total kinetic energy K of the body is then given by the sum of the kinetic energies of individual particles,

$$K = \sum_{i=1}^{n} k_i = \frac{1}{2}\sum_{i=1}^{n}(m_i r_i^2\omega^2)$$

Here n is the number of particles in the body. Note ω is the same for all particles. Hence, taking ω out of the sum,

$$K = \frac{1}{2}\omega^2\left(\sum_{i=1}^{n} m_i r_i^2\right)$$

We define a new parameter characterising the rigid body, called the moment of inertia I, given by

$$\boxed{I = \sum_{i=1}^{n} m_i r_i^2} \qquad (7.34)$$

With this definition,

$$\boxed{K = \frac{1}{2}I\omega^2} \qquad (7.35)$$

Note that the parameter I is independent of the magnitude of the angular velocity. It is a characteristic of the rigid body and the axis about which it rotates.

Compare Eq. (7.35) for the kinetic energy of a rotating body with the expression for the kinetic energy of a body in linear (translational) motion,

$$K = \frac{1}{2}m\,v^2$$

Here, m is the mass of the body and v is its velocity. We have already noted the analogy between angular velocity ω (in respect of rotational motion about a fixed axis) and linear velocity v (in respect of linear motion). It is then evident that the parameter, moment of inertia I, is the desired rotational analogue of mass in linear motion. In rotation (about a fixed axis), the moment of inertia plays a similar role as mass does in linear motion. Imp

We now apply the definition Eq. (7.34), to calculate the moment of inertia in two simple cases.

(a) Consider a thin ring of radius R and mass M, rotating in its own plane around its centre