

examples and identify the fulcrum, the effort and effort arm, and the load and the load arm of the lever in each case.

You may easily show that the principle of moment holds even when the parallel forces F_1 and F_2 are not perpendicular, but act at some angle, to the lever.

7.8.2 Centre of gravity

Many of you may have the experience of balancing your notebook on the tip of a finger. Figure 7.24 illustrates a similar experiment that you can easily perform. Take an irregularshaped cardboard having mass M and a narrow tipped object like a pencil. You can locate by trial and error a point G on the cardboard where it can be balanced on the tip of the pencil. (The cardboard remains horizontal in this position.) This point of balance is the centre of gravity (CG) of the cardboard. The tip of the pencil provides a vertically upward force due to which the cardboard is in mechanical equilibrium. As shown in the Fig. 7.24, the reaction of the tip is equal and opposite to Mg and hence the cardboard is in translational equilibrium. It is also in rotational equilibrium; if it were not so, due to the unbalanced torque it would tilt and fall. There are torques on the card board due to the forces of gravity like $m_1 \mathbf{g}, m_2 \mathbf{g} \dots$ etc, acting on the individual particles that make up the cardboard.



Fig. **7.24** Balancing a cardboard on the tip of a pencil. The point of support, G, is the centre of gravity.

The CG of the cardboard is so located that the total torque on it due to the forces $m_1 \mathbf{g}$, $m_2 \mathbf{g}$ etc. is zero.

If \mathbf{r}_i is the position vector of the *i*th particle of an extended body with respect to its CG, then the torque about the CG, due to the force of gravity on the particle is $\mathbf{\tau}_i = \mathbf{r}_i \times m_i \mathbf{g}$. The total gravitational torque about the CG is zero, i.e.

 $\mathbf{\tau}_q = \sum \mathbf{\tau}_i = \sum \mathbf{r}_i \times m_i \mathbf{g} = \mathbf{0}$

We may therefore, define the CG of a body \mathcal{F} bef \mathcal{F} as that point where the total gravitational torque $\mathcal{F}_{\mathcal{C}}$ on the body is zero.

We notice that in Eq. (7.33), **g** is the same for all particles, and hence it comes out of the summation. This gives, since **g** is non-zero,

 $\sum m_i \mathbf{r}_i = \mathbf{0}$. Remember that the position vectors

(**r**) are taken with respect to the CG. Now, in accordance with the reasoning given below Eq. (7.4a) in Sec. 7.2, if the sum is zero, the origin must be the centre of mass of the body. Thus, the centre of gravity of the body coincides with the centre of mass in uniform gravity or gravity-free space. We note that this is true because the body being small, **g** does not



Fig. 7.25 Determining the centre of gravity of a body of irregular shape. The centre of gravity G lies on the vertical AA₁ through the point of suspension of the body A.

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(7.33)