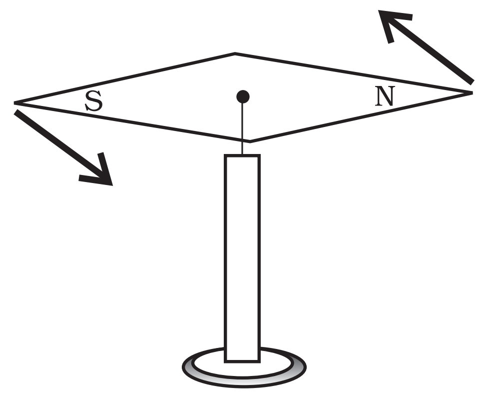


Fig. 7.21(b) *The Earth's magnetic field exerts equal and opposite forces on the poles of a compass needle. These two forces form a couple.*

Remember

Example 7.7 Show that moment of a couple does not depend on the point about which you take the moments.

Answer

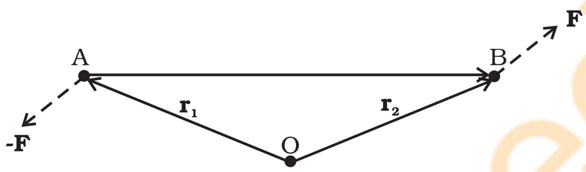


Fig. 7.22

Consider a couple as shown in Fig. 7.22 acting on a rigid body. The forces **F** and -**F** act respectively at points B and A. These points have position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ with respect to origin O. Let us take the moments of the forces about the origin.

The moment of the couple = sum of the moments of the two forces making the couple

$= \mathbf{r}_1 \times (-\mathbf{F}) + \mathbf{r}_2 \times \mathbf{F}$

$= \mathbf{r}_2 \times \mathbf{F} - \mathbf{r}_1 \times \mathbf{F}$

$= (\mathbf{r}_2 - \mathbf{r}_1) \times \mathbf{F}$

But $\mathbf{r}_1 + \mathbf{AB} = \mathbf{r}_2$, and hence $\mathbf{AB} = \mathbf{r}_2 - \mathbf{r}_1$.

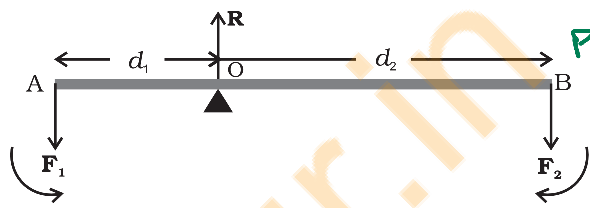
The moment of the couple, therefore, is $\mathbf{AB} \times \mathbf{F}$.

Clearly this is independent of the origin, the point about which we took the moments of the forces. ◀

7.8.1 Principle of moments

What is an ideal lever?

An ideal lever is essentially a light (i.e. of negligible mass) rod pivoted at a point along its length. This point is called the fulcrum. A see-saw on the children's playground is a typical example of a lever. Two forces F_1 and F_2, parallel to each other and usually perpendicular to the lever, as shown here, act on the lever at distances d_1 and d_2 respectively from the fulcrum as shown in Fig. 7.23.



Point-O is fulcrum.

Fig. 7.23

The lever is a system in mechanical equilibrium. Let **R** be the reaction of the support at the fulcrum; **R** is directed opposite to the forces F_1 and F_2. For translational equilibrium,

$R - F_1 - F_2 = 0$ → $R = F_1 + F_2$ (i)

For considering rotational equilibrium we take the moments about the fulcrum; the sum of moments must be zero,

$d_1F_1 - d_2F_2 = 0$ → $d_1F_1 = d_2F_2$ (ii)

Normally the anticlockwise (clockwise) moments are taken to be positive (negative). Note R acts at the fulcrum itself and has zero moment about the fulcrum.

In the case of the lever force F_1 is usually some weight to be lifted. It is called the *load* and its distance from the fulcrum d_1 is called the *load arm*. Force F_2 is the *effort* applied to lift the load; distance d_2 of the effort from the fulcrum is the *effort arm.*

Eq. (ii) can be written as

$d_1F_1 = d_2F_2$ (7.32a)

or load arm × load = effort arm × effort

The above equation expresses the principle of moments for a lever. Incidentally the ratio F_1/F_2 is called the Mechanical Advantage (M.A.);

$$\boxed{\text{M.A.} = \frac{F_1}{F_2} = \frac{d_2}{d_1}} \quad (7.32b)$$

If the effort arm d_2 is larger than the load arm, the mechanical advantage is greater than one. Mechanical advantage greater than one means that a small effort can be used to lift a large load. Imp There are several examples of a lever around you besides the see-saw. The beam of a balance is a lever. Try to find more such