

Eq. (7.31a) and (7.31b) give six independent conditions to be satisfied for mechanical equilibrium of a rigid body. In a number of problems all the forces acting on the body are coplanar. Then we need only three conditions to be satisfied for mechanical equilibrium. Two of these conditions correspond to translational equilibrium; the sum of the components of the forces along any two perpendicular axes in the plane must be zero. The third condition corresponds to rotational equilibrium. The sum of the components of the torques along any axis perpendicular to the plane of the forces must be zero.

The conditions of equilibrium of a rigid body may be compared with those for a particle, which we considered in earlier chapters. Since consideration of rotational motion does not apply to a particle, only the conditions for translational equilibrium (Eq. 7.30 a) apply to a particle. Thus, for equilibrium of a particle the vector sum of all the forces on it must be zero. Since all these forces act on the single particle, they must be concurrent. Equilibrium under concurrent forces was discussed in the earlier chapters.

A body may be in partial equilibrium, i.e., it may be in translational equilibrium and not in rotational equilibrium, or it may be in rotational equilibrium and not in translational equilibrium.

Consider a light (i.e. of negligible mass) rod (AB) as shown in Fig. 7.20(a). At the two ends (A and B) of which two parallel forces, both equal in magnitude and acting along same direction are applied perpendicular to the rod.

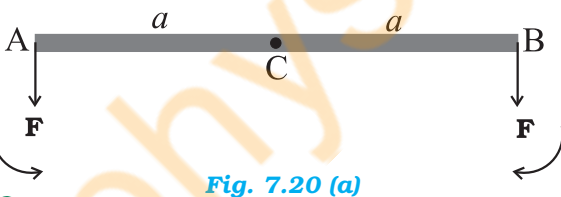


Fig. 7.20 (a)

Let C be the midpoint of AB, CA = CB = a. the moment of the forces at A and B will both be equal in magnitude (aF), but opposite in sense as shown. The net moment on the rod will be zero. The system will be in rotational equilibrium, but it will not be in translational equilibrium; $\sum \mathbf{F} \neq \mathbf{0}$

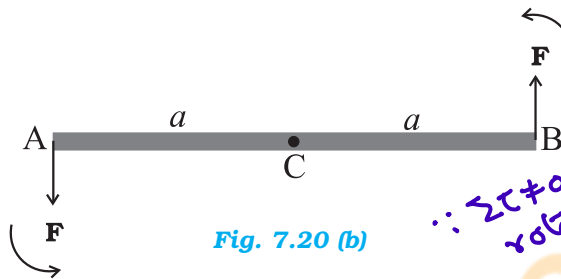


Fig. 7.20 (b)

The force at B in Fig. 7.20(a) is reversed in Fig. 7.20(b). Thus, we have the same rod with two forces of equal magnitude but acting in opposite directions applied perpendicular to the rod, one at end A and the other at end B. Here the moments of both the forces are equal, but they are not opposite; they act in the same sense and cause anticlockwise rotation of the rod. The total force on the body is zero; so the body is in translational equilibrium; but it is not in rotational equilibrium. Although the rod is not fixed in any way, it undergoes pure rotation (i.e. rotation without translation).

A pair of forces of equal magnitude but acting in opposite directions with different lines of action is known as a **couple** or **torque**. A couple produces rotation without translation.

When we open the lid of a bottle by turning it, our fingers are applying a couple to the lid [Fig. 7.21(a)]. Another known example is a compass needle in the earth's magnetic field as shown in the Fig. 7.21(b). The earth's magnetic field exerts equal forces on the north and south poles. The force on the North Pole is towards the north, and the force on the South Pole is toward the south. Except when the needle points in the north-south direction; the two forces do not have the same line of action. Thus there is a **couple** acting on the needle due to the earth's magnetic field.



Fig. 7.21(a) Our fingers apply a couple to turn the lid.

$\sum F_x = 0$
 $\sum F_y = 0$
 $\sum \tau = 0$
about any point

$\therefore \sum \mathbf{F} = 0$
 \rightarrow rod is in translational equilibrium.
 $\therefore \sum \tau \neq 0$, rod is not in rotational equilibrium.

Partial equilibrium

Def of force couple

$\therefore \sum \mathbf{F} \neq 0$
There is no translational equilibrium
But, as $\sum \tau = 0$ (about C)
 \rightarrow the rod is in rotational equilibrium