Answer Let the particle with velocity **v** be at point P at some instant *t*. We want to calculate the angular momentum of the particle about an arbitrary point O.

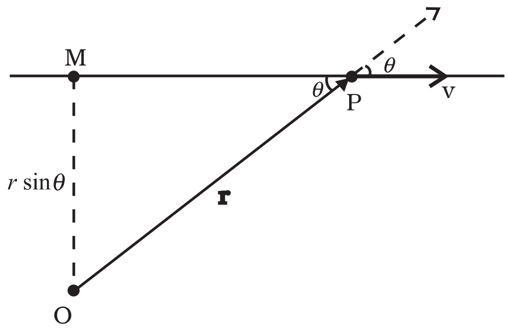

Fig 7.19

The angular momentum is $\mathbf{l} = \mathbf{r} \times m\mathbf{v}$. Its magnitude is $mvr \sin\theta$, where θ is the angle between **r** and **v** as shown in Fig. 7.19. Although the particle changes position with time, the line of direction of **v** remains the same and hence $OM = r \sin\theta$. is a constant.

Further, the direction of **l** is perpendicular to the plane of **r** and **v**. It is into the page of the figure.This direction does not change with time.

Thus, **l** remains the same in magnitude and direction and is therefore conserved. Is there any external torque on the particle?

7.8 EQUILIBRIUM OF A RIGID BODY

We are now going to concentrate on the motion of rigid bodies rather than on the motion of general systems of particles.

We shall recapitulate what effect the external forces have on a rigid body. (Henceforth we shall omit the adjective 'external' because unless stated otherwise, we shall deal with only external forces and torques.) The forces change the translational state of the motion of the rigid body, i.e. they change its total linear momentum in accordance with Eq. (7.17). But this is not the only effect the forces have. The total torque on the body may not vanish. Such a torque changes the rotational state of motion of the rigid body, i.e. it changes the total angular momentum of the body in accordance with Eq. (7.28 b).

what is Mech- equilibrium?

A rigid body is said to be in mechanical equilibrium, if both its linear momentum and angular momentum are not changing with time, or equivalently, the body has neither linear acceleration nor angular acceleration. This means

(1) the total force, i.e. the vector sum of the forces, on the rigid body is zero;

$$\mathbf{F}_1 + \mathbf{F}_2 + \ldots + \mathbf{F}_n = \sum_{i=1}^{n} \mathbf{F}_i = \mathbf{0} \qquad (7.30a)$$

$\sum\vec{F}=0 \Rightarrow \sum F_x=0$, $\sum F_y=0$, $\sum F_z=0$

If the total force on the body is zero, then the total linear momentum of the body does not change with time. Eq. (7.30a) gives the condition for the translational equilibrium of the body.

(2) The total torque, i.e. the vector sum of the torques on the rigid body is zero,

$\sum\vec{\tau}=0$ about any point

$$\boldsymbol{\tau}_1 + \boldsymbol{\tau}_2 + \ldots + \boldsymbol{\tau}_n = \sum_{i=1}^{n} \boldsymbol{\tau}_i = \mathbf{0} \qquad (7.30b)$$

If the total torque on the rigid body is zero, the total angular momentum of the body does not change with time. Eq. (7.30 b) gives the condition for the rotational equilibrium of the body.

One may raise a question, whether the rotational equilibrium condition [Eq. 7.30(b)] remains valid, if the origin with respect to which the torques are taken is shifted. One can show that if the translational equilibrium condition [Eq. 7.30(a)] holds for a rigid body, then such a shift of origin does not matter, i.e. the rotational equilibrium condition is independent of the location of the origin about which the torques are taken. Example 7.7 gives a proof of this result in a special case of a couple, i.e. two forces acting on a rigid body in translational equilibrium. The generalisation of this result to n forces is left as an exercise.

Eq. (7.30a) and Eq. (7.30b), both, are vector equations. They are equivalent to three scalar equations each. Eq. (7.30a) corresponds to

$$\sum_{i=1}^{n} F_{ix} = 0, \quad \sum_{i=1}^{n} F_{iy} = 0 \text{ and } \sum_{i=1}^{n} F_{iz} = 0 \qquad (7.31a)$$

where F_{ix}, F_{iy} and F_{iz} are respectively the x, y and z components of the forces $\mathbf{F}_i$. Similarly, Eq. (7.30b) is equivalent to three scalar equations

$$\sum_{i=1}^{n} \tau_{ix} = 0, \quad \sum_{i=1}^{n} \tau_{iy} = 0 \text{ and } \sum_{i=1}^{n} \tau_{iz} = 0 \qquad (7.31b)$$

where τ_{ix}, τ_{iy} and τ_{iz} are respectively the x, y and z components of the torque $\boldsymbol{\tau}_i$.