where $\boldsymbol{\tau}_i$ is the torque acting on the i^{th} particle;

$\boldsymbol{\tau}_i = \mathbf{r}_i \times \mathbf{F}_i$

The force $\mathbf{F}_i$ on the i^{th} particle is the vector sum of external forces $\mathbf{F}_i^{ext}$ acting on the particle and the internal forces $\mathbf{F}_i^{int}$ exerted on it by the other particles of the system. We may therefore separate the contribution of the external and the internal forces to the total torque $\boldsymbol{\tau} = \sum_i \boldsymbol{\tau}_i = \sum_i \mathbf{r}_i \times \mathbf{F}_i$ as

$$\boxed{\boldsymbol{\tau} = \boldsymbol{\tau}_{ext} + \boldsymbol{\tau}_{int}},$$

where $\boldsymbol{\tau}_{ext} = \sum_i \mathbf{r}_i \times \mathbf{F}_i^{ext}$

and $\boldsymbol{\tau}_{int} = \sum_i \mathbf{r}_i \times \mathbf{F}_i^{int}$

We shall assume not only Newton's third law of motion, i.e. the forces between any two particles of the system are equal and opposite, but also that these forces are directed along the line joining the two particles. In this case the contribution of the internal forces to the total torque on the system is zero, since the torque resulting from each action-reaction pair of forces is zero. We thus have, $\boldsymbol{\tau}_{int} = \mathbf{0}$ and therefore $\boldsymbol{\tau} = \boldsymbol{\tau}_{ext}$.

Since $\boldsymbol{\tau} = \sum \boldsymbol{\tau}_i$, it follows from Eq. (7.28a) that

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$$\boxed{\frac{d\mathbf{L}}{dt} = \boldsymbol{\tau}_{ext}} \qquad (7.28\text{ b})$$

Thus, **the time rate of the total angular momentum of a system of particles about a point** (taken as the origin of our frame of reference) **is equal to the sum of the external torques** (i.e. the torques due to external forces) **acting on the system taken about the same point**. Eq. (7.28 b) is the generalisation of the single particle case of Eq. (7.23) to a system of particles. Note that when we have only one particle, there are no internal forces or torques. Eq.(7.28 b) is the rotational analogue of

$$\frac{d\mathbf{P}}{dt} = \mathbf{F}_{ext} \qquad (7.17)$$

Note that like Eq.(7.17), Eq.(7.28b) holds good for any system of particles, whether it is a rigid body or its individual particles have all kinds of internal motion.

Conservation of angular momentum

If $\boldsymbol{\tau}_{ext} = \mathbf{0}$, Eq. (7.28b) reduces to

$$\frac{d\mathbf{L}}{dt} = 0$$

or $\quad \mathbf{L} = \text{constant}. \qquad (7.29\text{a})$

Thus, if the total external torque on a system of particles is zero, then the total angular momentum of the system is conserved, i.e. remains constant. Eq. (7.29a) is equivalent to three scalar equations,

statement of Principle of conservation of angular momentum

$$L_x = K_1,\ L_y = K_2 \text{ and } L_z = K_3 \qquad (7.29\text{ b})$$

Here K_1, K_2 and K_3 are constants; L_x, L_y and L_z are the components of the total angular momentum vector $\mathbf{L}$ along the *x,y* and *z* axes respectively. The statement that the total angular momentum is conserved means that each of these three components is conserved.

If $\vec{\tau}_{ext} = 0$

L_x = constant
L_y = constant
L_z = const

Eq. (7.29a) is the rotational analogue of Eq. (7.18a), i.e. the conservation law of the total linear momentum for a system of particles. Like Eq. (7.18a), it has applications in many practical situations. We shall look at a few of the interesting applications later on in this chapter.

▶ ***Example 7.5*** Find the torque of a force $7\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ about the origin. The force acts on a particle whose position vector is $\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$.

Answer Here $\mathbf{r} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$

and $\mathbf{F} = 7\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$.

We shall use the determinant rule to find the torque $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

$$\tau = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -1 & 1 \\ 7 & 3 & -5 \end{vmatrix} = (5-3)\hat{\mathbf{i}} - (-5-7)\hat{\mathbf{j}} + (3-(-7))\hat{\mathbf{k}}$$

or $\boldsymbol{\tau} = 2\hat{\mathbf{i}} + 12\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$ ◀

▶ ***Example 7.6*** Show that the angular momentum about any point of a single particle moving with constant velocity remains constant throughout the motion.

Remember this result