Now $\boldsymbol{\omega} \times \mathbf{r} = \boldsymbol{\omega} \times \mathbf{OP} = \boldsymbol{\omega} \times (\mathbf{OC} + \mathbf{CP})$

But $\boldsymbol{\omega} \times \mathbf{OC} = 0$ as $\boldsymbol{\omega}$ is along $\mathbf{OC}$

Hence $\boldsymbol{\omega} \times \mathbf{r} = \boldsymbol{\omega} \times \mathbf{CP}$

The vector $\boldsymbol{\omega} \times \mathbf{CP}$ is perpendicular to $\boldsymbol{\omega}$, i.e. to the z-axis and also to **CP**, the radius of the circle described by the particle at P. It is therefore, along the tangent to the circle at P. Also, the magnitude of $\boldsymbol{\omega} \times \mathbf{CP}$ is ω (CP) since $\boldsymbol{\omega}$ and **CP** are perpendicular to each other. We shall denote **CP** by $\mathbf{r}_{\perp}$ and not by $\mathbf{r}$, as we did earlier.

Thus, $\boldsymbol{\omega} \times \mathbf{r}$ is a vector of magnitude $\omega r_{\perp}$ and is along the tangent to the circle described by the particle at P. The linear velocity vector **v** at P has the same magnitude and direction. Thus,

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r} \tag{7.20}$$

In fact, the relation, Eq. (7.20), holds good even for rotation of a rigid body with one point fixed, such as the rotation of the top [Fig. 7.6(a)]. In this case **r** represents the position vector of the particle with respect to the fixed point taken as the origin.

Note

We note that **for rotation about a fixed axis, the direction of the vector ω does not change with time. Its magnitude may, however, change from instant to instant. For the more general rotation, both the magnitude and the direction of ω may change from instant to instant.**

Direction of ω remains same unless the sence of rotation changes.

This statement is valid for the motion like precession of a top.

7.6.1 Angular acceleration

You may have noticed that we are developing the study of rotational motion along the lines of the study of translational motion with which we are already familiar. Analogous to the kinetic variables of linear displacement (**s**) and velocity (**v**) in translational motion, we have angular displacement (**θ**) and angular velocity (**ω**) in rotational motion. It is then natural to define in rotational motion the concept of angular acceleration in analogy with linear acceleration defined as the time rate of change of velocity in translational motion. We define angular acceleration **α** as the time rate of change of angular velocity; Thus,

$$\boldsymbol{\alpha} = \frac{d\boldsymbol{\omega}}{dt} \tag{7.21}$$

Def. of angular acceleration

If the axis of rotation is fixed, the direction of **ω** and hence, that of **α** is fixed. In this case the vector equation reduces to a scalar equation

$$\alpha = \frac{d\omega}{dt} \tag{7.22}$$

7.7 TORQUE AND ANGULAR MOMENTUM

In this section, we shall acquaint ourselves with two physical quantities (torque and angular momentum) which are defined as vector products of two vectors. These as we shall see, are especially important in the discussion of motion of systems of particles, particularly rigid bodies.

7.7.1 Moment of force (Torque)

We have learnt that the motion of a rigid body, in general, is a combination of rotation and translation. If the body is fixed at a point or along a line, it has only rotational motion. We know that force is needed to change the translational state of a body, i.e. to produce linear acceleration. We may then ask, what is the analogue of force in the case of rotational motion? To look into the question in a concrete situation let us take the example of opening or closing of a door. A door is a rigid body which can rotate about a fixed vertical axis passing through the hinges. What makes the door rotate? It is clear that unless a force is applied the door does not rotate. But any force does not do the job. A force applied to the hinge line cannot produce any rotation at all, whereas a force of given magnitude applied at right angles to the door at its outer edge is most effective in producing rotation. It is not the force alone, but how and where the force is applied is important in rotational motion.

The rotational analogue of force in linear motion is **moment of force**. It is also referred to as **torque** or **couple**. (We shall use the words moment of force and torque interchangeably.) We shall first define the moment of force for the special case of a single particle. Later on we shall extend the concept to systems of particles including rigid bodies. We shall also relate it to a change in the state of rotational motion, i.e. is angular acceleration of a rigid body.

Torque plays the same role in rotational motion as force plays in translational motion.