* If the body is rotating anticlockwise, the direction of ω is outward, along the axis of rotation and vice-versa.

(2)

a circle with a centre C on the axis. The radius of the circle is r, the perpendicular distance of the point P from the axis. We also show the linear velocity vector **v** of the particle at P. It is along the tangent at P to the circle.

Let P′ be the position of the particle after an interval of time Δt (Fig. 7.16). The angle PCP′ describes the angular displacement $\Delta\theta$ of the particle in time Δt. The average angular velocity of the particle over the interval Δt is $\Delta\theta/\Delta t$. As Δt tends to zero (i.e. takes smaller and smaller values), the ratio $\Delta\theta/\Delta t$ approaches a limit which is the instantaneous angular velocity $d\theta/dt$ of the particle at the position P. We denote the **instantaneous angular velocity** by ω (the Greek letter omega). We know from our study of circular motion that the magnitude of linear velocity v of a particle moving in a circle is related to the angular velocity of the particle ω by the simple relation $v = \omega r$, where r is the radius of the circle.

$\omega_{av} = \frac{\Delta\theta}{\Delta t}$

$\omega = \lim_{\Delta t \to 0}\left(\frac{\Delta\theta}{\Delta t}\right)$

or $\omega_{inst} = \frac{d\theta}{dt}$

$v = \omega r$

We observe that at any given instant the relation $v = \omega r$ applies to all particles of the rigid body. Thus for a particle at a perpendicular distance r_i from the fixed axis, the linear velocity at a given instant v_i is given by

$v_i = \omega r_i$

$$v_i = \omega r_i \qquad (7.19)$$

The index i runs from 1 to n, where n is the total number of particles of the body.

For particles on the axis, $r = 0$, and hence $v = \omega r = 0$. Thus, particles on the axis are stationary. This verifies that the axis is *fixed*.

Note that we use the same angular velocity ω for all the particles. **We therefore, refer to ω as the angular velocity of the whole body.**

v. Imp

We have characterised pure translation of a body by all parts of the body having the same velocity at any instant of time. Similarly, we may characterise pure rotation by all parts of the body having the same angular velocity at any instant of time. Note that this characterisation of the rotation of a rigid body about a fixed axis is **just another way** of saying as in Sec. 7.1 that each particle of the body moves in a circle, which lies in a plane perpendicular to the axis and has the centre on the axis.

In our discussion so far the angular velocity appears to be a scalar. In fact, it is a vector. We shall not justify this fact, but we shall accept it. For rotation about a fixed axis, the angular velocity vector lies along the axis of rotation, and points out in the direction in which a right handed screw would advance, if the head of the screw is rotated with the body. (See Fig. 7.17a).

The magnitude of this vector is $\omega = d\theta/dt$ referred as above.

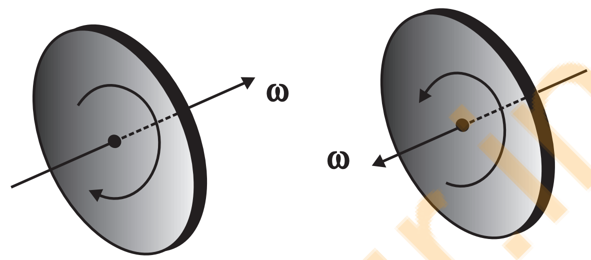

Fig. 7.17 (a) *If the head of a right handed screw rotates with the body, the screw advances in the direction of the angular velocity **ω**. If the sense (clockwise or anticlockwise) of rotation of the body changes, so does the direction of **ω**.*

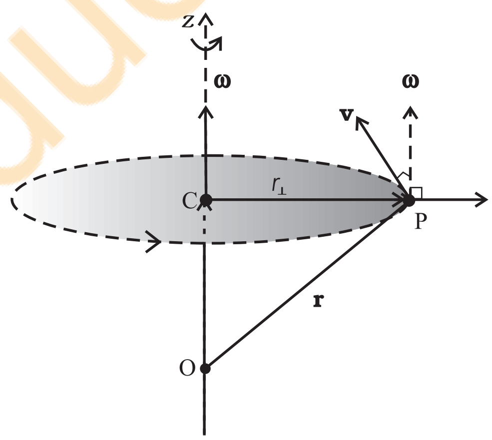

Fig. 7.17 (b) *The angular velocity vector **ω** is directed along the fixed axis as shown. The linear velocity of the particle at P is **v** = **ω** × **r**. It is perpendicular to both **ω** and **r** and is directed along the tangent to the circle described by the particle.*

$\vec{v} = \vec{\omega} \times \vec{r}$

We shall now look at what the vector product $\boldsymbol{\omega} \times \mathbf{r}$ corresponds to. Refer to Fig. 7.17(b) which is a part of Fig. 7.16 reproduced to show the path of the particle P. The figure shows the vector **ω** directed along the fixed (z–) axis and also the position vector **r** = **OP** of the particle at P of the rigid body with respect to the origin O. Note that the origin is chosen to be on the axis of rotation.