the value of x between $-A$ and A, the acceleration $a(t)$ is always directed towards the centre.
For simplicity, let us put $\phi = 0$ and write the expression for $x(t)$, $v(t)$ and $a(t)$

$x(t) = A\cos\omega t$, $v(t) = -\omega A\sin\omega t$, $a(t) = -\omega^2 A\cos\omega t$

The corresponding plots are shown in Fig. 14.13. All quantities vary sinusoidally with time; only their maxima differ and the different plots differ in phase. x varies between $-A$ to A; $v(t)$ varies from $-\omega A$ to ωA and $a(t)$ from $-\omega^2 A$ to $\omega^2 A$. With respect to displacement plot, velocity plot has a phase difference of $\pi/2$ and acceleration plot has a phase difference of π.

$x = A\cos\omega t$

$v = -A\omega \sin\omega t$

$a = -\omega^2 x$

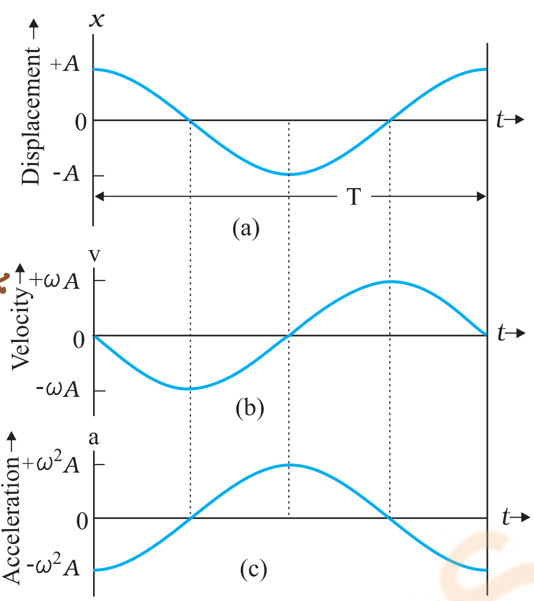

Fig. 14.13 *Displacement, velocity and acceleration of a particle in simple harmonic motion have the same period T, but they differ in phase*

> **Example 14.5** A body oscillates with SHM according to the equation (in SI units),
>
> $x = 5\cos[2\pi t + \pi/4]$.
>
> At $t = 1.5$ s, calculate the (a) displacement, (b) speed and (c) acceleration of the body.

Answer The angular frequency ω of the body $= 2\pi\ \text{s}^{-1}$ and its time period $T = 1$ s.

At $t = 1.5$ s

(a) displacement $= (5.0\ \text{m})\cos[(2\pi\ \text{s}^{-1}) \times 1.5\ \text{s} + \pi/4]$
$= (5.0\ \text{m})\cos[(3\pi + \pi/4)]$
$= -5.0 \times 0.707$ m
$= -3.535$ m

(b) Using Eq. (14.9), the speed of the body
$= -(5.0\ \text{m})(2\pi\ \text{s}^{-1})\sin[(2\pi\ \text{s}^{-1}) \times 1.5\ \text{s} + \pi/4]$
$= -(5.0\ \text{m})(2\pi\ \text{s}^{-1})\sin[(3\pi + \pi/4)]$
$= 10\pi \times 0.707\ \text{m s}^{-1}$
$= 22\ \text{m s}^{-1}$

(c) Using Eq.(14.10), the acceleration of the body
$= -(2\pi\ \text{s}^{-1})^2 \times$ displacement
$= -(2\pi\ \text{s}^{-1})^2 \times (-3.535\ \text{m})$
$= 140\ \text{m s}^{-2}$

14.6 FORCE LAW FOR SIMPLE HARMONIC MOTION

Using Newton's second law of motion, and the expression for acceleration of a particle undergoing SHM (Eq. 14.11), the force acting on a particle of mass m in SHM is

$$F(t) = ma$$

$$\boxed{F(t) = -m\omega^2 x(t)}$$

i.e., $F(t) = -k\,x(t)$ (14.13)

where $k = m\omega^2$ (14.14a)

or $\omega = \sqrt{\dfrac{k}{m}}$ (14.14b)

Like acceleration, force is always directed towards the mean position—hence it is sometimes called the restoring force in SHM. To summarise the discussion so far, simple harmonic motion can be defined in two equivalent ways, either by Eq. (14.4) for displacement or by Eq. (14.13) that gives its force law. Going from Eq. (14.4) to Eq. (14.13) required us to differentiate two times. Likewise, by integrating the force law Eq. (14.13) two times, we can get back Eq. (14.4).

Note that the force in Eq. (14.13) is linearly proportional to $x(t)$. A particle oscillating under such a force is, therefore, calling a linear harmonic oscillator. In the real world, the force may contain small additional terms proportional to x^2, x^3, etc. These then are called non-linear oscillators.

> **Example 14.6** Two identical springs of spring constant k are attached to a block of mass m and to fixed supports as shown in Fig. 14.14. Show that when the mass is displaced from its equilibrium position on either side, it executes a simple harmonic motion. Find the period of oscillations.