(b) In this case at $t = 0$, OP makes an angle of $90^\circ = \frac{\pi}{2}$ with the x-axis. After a time t, it covers an angle of $\frac{2\pi}{T}t$ in the clockwise sense and makes an angle of $\left(\frac{\pi}{2} - \frac{2\pi}{T}t\right)$ with the x-axis. The projection of OP on the x-axis at time t is given by

$$x(t) = B\cos\left(\frac{\pi}{2} - \frac{2\pi}{T}t\right)$$

$$= B\sin\left(\frac{2\pi}{T}t\right)$$

For $T = 30$ s,

$$x(t) = B\sin\left(\frac{\pi}{15}t\right)$$

Writing this as $x(t) = B\cos\left(\frac{\pi}{15}t - \frac{\pi}{2}\right)$, and comparing with Eq. (14.4). We find that this represents a SHM of amplitude B, period 30 s, and an initial phase of $-\frac{\pi}{2}$.

14.5 VELOCITY AND ACCELERATION IN SIMPLE HARMONIC MOTION

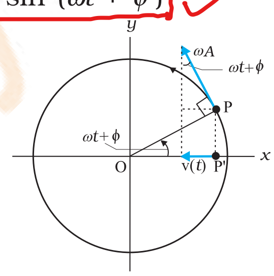
The speed of a particle v in uniform circular motion is its angular speed ω times the radius of the circle A.

$$v = \omega A \qquad (14.8)$$

The direction of velocity $\vec{\mathbf{v}}$ at a time t is along the tangent to the circle at the point where the particle is located at that instant. From the geometry of Fig. 14.11, it is clear that the velocity of the projection particle P′ at time t is

component of velocity of particle P along x-axis

$$\boxed{v(t) = -\omega A\sin(\omega t + \phi)} \qquad (14.9)$$



(component) **Fig. 14.11** *The velocity, v (t), of the particle P′ is the projection of the velocity* $\vec{\mathbf{v}}$ *of the reference particle, P.*

where the negative sign shows that $v(t)$ has a direction opposite to the positive direction of x-axis. Eq. (14.9) gives the instantaneous velocity of a particle executing SHM, where displacement is given by Eq. (14.4). We can, of course, obtain this equation without using geometrical argument, directly by differentiating (Eq. 14.4) with respect of t:

$$v(t) = \frac{\mathrm{d}}{\mathrm{d}t}x(t) \qquad (14.10)$$

The method of reference circle can be similarly used for obtaining instantaneous acceleration of a particle undergoing SHM. We know that the centripetal acceleration of a particle P in uniform circular motion has a magnitude v^2/A or $\omega^2 A$, and it is directed towards the centre i.e., the direction is along PO. The instantaneous acceleration of the projection particle P′ is then (See Fig. 14.12)

$$a(t) = -\omega^2 A\cos(\omega t + \phi)$$

$$\boxed{a(t) = -\omega^2 x(t)} \qquad (14.11)$$

component of acceleration $\vec{a}$ along x-axis

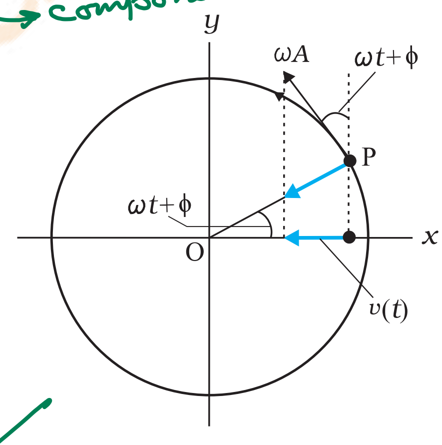


Fig. 14.12 *The acceleration, a(t), of the particle P′ is the projection of the acceleration* **a** *of the reference particle P.*

Eq. (14.11) gives the acceleration of a particle in SHM. The same equation can again be obtained directly by differentiating velocity $v(t)$ given by Eq. (14.9) with respect to time:

$$a(t) = \frac{\mathrm{d}}{\mathrm{d}t}v(t) \qquad (14.12)$$

We note from Eq. (14.11) the important property that acceleration of a particle in SHM is proportional to displacement. For $x(t) > 0$, $a(t) < 0$ and for $x(t) < 0$, $a(t) > 0$. Thus, whatever