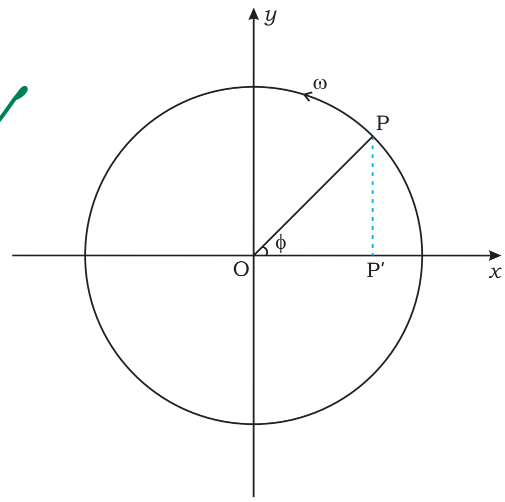


Fig. 14.10

will make an angle of $\omega t + \phi$ with the +ve x-axis. Next, consider the projection of the position vector OP on the x-axis. This will be OP′. The position of P′ on the x-axis, as the particle P moves on the circle, is given by

$$x(t) = A \cos (\omega t + \phi)$$

Imp

which is the defining equation of SHM. This shows that if P moves uniformly on a circle, its projection P′ on a diameter of the circle executes SHM. The particle P and the circle on which it moves are sometimes referred to as the *reference particle* and the *reference circle*, respectively.

We can take projection of the motion of P on any diameter, say the y-axis. In that case, the displacement $y(t)$ of P′ on the y-axis is given by

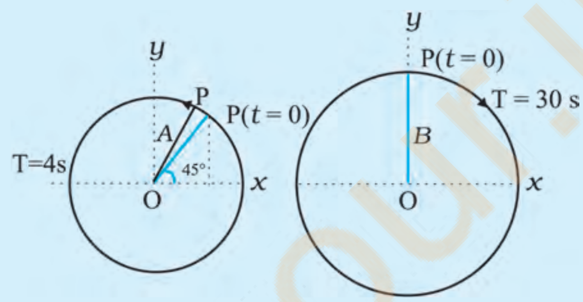
$$y = A \sin (\omega t + \phi)$$

which is also an SHM of the same amplitude as that of the projection on x-axis, but differing by a phase of $\pi/2$.

In spite of this connection between circular motion and SHM, the force acting on a particle in linear simple harmonic motion is very different from the centripetal force needed to keep a particle in uniform circular motion.

Imp

Example 14.4 The figure given below depicts two circular motions. The radius of the circle, the period of revolution, the initial position and the sense of revolution are indicated in the figures. Obtain the simple harmonic motions of the x-projection of the radius vector of the rotating particle P in each case.



Answer

(a) At $t = 0$, OP makes an angle of $45° = \pi/4$ rad with the (positive direction of) x-axis. After time t, it covers an angle $\frac{2\pi}{T}t$ in the anticlockwise sense, and makes an angle of $\frac{2\pi}{T}t + \frac{\pi}{4}$ with the x-axis.

The projection of OP on the x-axis at time t is given by,

$$x(t) = A \cos\left(\frac{2\pi}{T}t + \frac{\pi}{4}\right)$$

For $T = 4$ s,

$$x(t) = A \cos\left(\frac{2\pi}{4}t + \frac{\pi}{4}\right)$$

which is a SHM of amplitude A, period 4 s, and an initial phase* $= \frac{\pi}{4}$.

* *The natural unit of angle is radian, defined through the ratio of arc to radius. Angle is a dimensionless quantity. Therefore it is not always necessary to mention the unit 'radian' when we use π, its multiples or submultiples. The conversion between radian and degree is not similar to that between metre and centimetre or mile. If the argument of a trigonometric function is stated without units, it is understood that the unit is radian. On the other hand, if degree is to be used as the unit of angle, then it must be shown explicitly. For example, sin(15°) means sine of 15 degree, but sin(15) means sine of 15 radians. Hereafter, we will often drop 'rad' as the unit, and it should be understood that whenever angle is mentioned as a numerical value, without units, it is to be taken as radians.*