Finally, the quantity ω can be seen to be related to the period of motion T. Taking, for simplicity, $\phi = 0$ in Eq. (14.4), we have

$$x(t) = A \cos \omega t \tag{14.5}$$

Since the motion has a period T, $x(t)$ is equal to $x(t + T)$. That is,

$$A \cos \omega t = A \cos \omega (t + T) \tag{14.6}$$

Now the cosine function is periodic with period 2π, i.e., it first repeats itself when the argument changes by 2π. Therefore,

$$\omega(t + T) = \omega t + 2\pi$$

that is $\omega = 2\pi/T$ (14.7)

ω is called the angular frequency of SHM. Its S.I. unit is radians per second. Since the frequency of oscillations is simply $1/T$, ω is 2π times the frequency of oscillation. Two simple harmonic motions may have the same A and ϕ, but different ω, as seen in Fig. 14.8. In this plot the curve (b) has half the period and twice the frequency of the curve (a).

$T_b = \frac{T_a}{2}$ and $\nu_b = 2\nu_a$

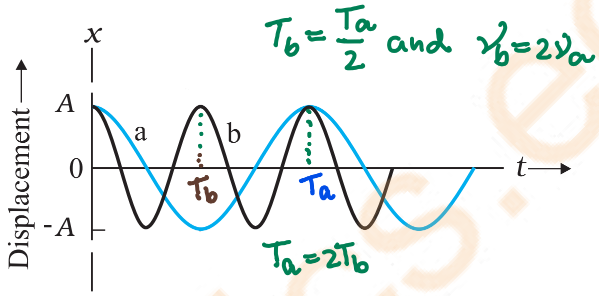

Fig. 14.8 Plots of Eq. (14.4) for $\phi = 0$ for two different periods.

Example 14.3 Which of the following functions of time represent (a) simple harmonic motion and (b) periodic but not simple harmonic? Give the period for each case.

(1) $\sin \omega t - \cos \omega t$
(2) $\sin^2 \omega t$

Answer

(a) $\sin \omega t - \cos \omega t$

$= \sin \omega t - \sin (\pi/2 - \omega t)$

$= 2 \cos (\pi/4) \sin (\omega t - \pi/4)$

$= \sqrt{2} \sin (\omega t - \pi/4)$

So, $A = \sqrt{2}$, $T = \frac{2\pi}{\omega}$ and $\varphi = -\pi/4$

This function represents a simple harmonic motion having a period $T = 2\pi/\omega$ and a phase angle $(-\pi/4)$ or $(7\pi/4)$

(b) $\sin^2 \omega t$

$= \frac{1}{2} - \frac{1}{2} \cos 2\omega t$

$\cos 2\theta = 1 - 2\sin^2\theta$ put $\theta = \omega t$

The function is periodic having a period $T = \pi/\omega$. It also represents a harmonic motion with the point of equilibrium occurring at ½ instead of zero.

The 2nd term $\frac{1}{2}\cos 2\omega t$ represent SHM with period $T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

Imp

14.4 SIMPLE HARMONIC MOTION AND UNIFORM CIRCULAR MOTION

In this section, we show that the projection of uniform circular motion on a diameter of the circle follows simple harmonic motion. A simple experiment (Fig. 14.9) helps us visualise this connection. Tie a ball to the end of a string and make it move in a horizontal plane about a fixed point with a constant angular speed. The ball would then perform a uniform circular motion in the horizontal plane. Observe the ball sideways or from the front, fixing your attention in the plane of motion. The ball will appear to execute to and fro motion along a horizontal line with the point of rotation as the midpoint. You could alternatively observe the shadow of the ball on a wall which is perpendicular to the plane of the circle. In this process what we are observing is the motion of the ball on a diameter of the circle normal to the direction of viewing.

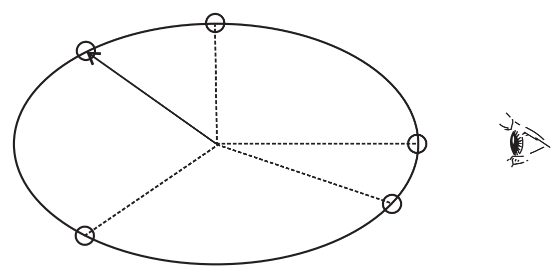

Fig. 14.9 Circular motion of a ball in a plane viewed edge-on is SHM.

Fig. 14.10 describes the same situation mathematically. Suppose a particle P is moving uniformly on a circle of radius A with angular speed ω. The sense of rotation is anticlockwise. The initial position vector of the particle, i.e., the vector $\overrightarrow{OP}$ at $t = 0$ makes an angle of ϕ with the positive direction of x-axis. In time t, it will cover a further angle ωt and its position vector